

a thesis presented by

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### Abstract

The South Indian mathematician Srinivasa Ramanujan (1887-1920) is famous for being an isolated auto-didact who rediscovered huge chunks of modern mathematics. However, closer examination of the narratives that present him as such reveals that ontological realist views of mathematics have biased authors towards loose interpretations of historical evidence. Examined here are a variety of sources, from a professional mathematician, to Indian nationalist hagiographers, to a postcolonial critic, to a popular biographer, and the way in which realism underpins these vastly different and often explicitly clashing interpretations of Ramanujan. In the process of critically reassessing these portrayals of Ramanujan, a more nuanced and insightful view of Ramanujan emerges that explains his peculiar set of mathematical skills and knowledge not as a miracle or mystery, but as the result of Ramanujan's relationship with the mathematical community around him.

#### Keywords

Ramanujan, Mathematics, Rediscovery, Genius, Realism, Constructivism, India, Kanigel, Nandy, Ranganathan, Hardy, Cambridge.

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## INTRODUCTION: Revisiting Ramanujan

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The above is from a facsimile copy of a page in Srinivasa Ramanujan's second notebook.<sup>1</sup> I imagine the reader will react as I did: first with shock and disbelief that something this strange could be true, next with bewilderment as to how somebody could imagine this, and third, with skepticism that this is in any way important or significant.

This is from a chapter of Ramanujan's 'elementary' results, not exceeding the level of exposure of perhaps high school math. There is no higher mathematics that makes this result any more comprehensible, or familiar, or for that matter useful. The mathematics involved here

<sup>&</sup>lt;sup>1</sup> Srinivasa Ramanujan Aiyangar, *Notebooks* Vol. 2 (Bombay: Tata Institute of Fundamental Research, 1957), 305. Cited hereafter as *Tata Institute* Vol. I or II, respectively.

is, beyond basic arithmetic operations only the concepts of a square root, sine, inverse tangent, and perhaps just the bare idea of an infinite series. Ramanujan has shown that some infinitely nested radicals are equal to certain trigonometric expressions. Here, he provided four variations, where differences in positive and negative signs within the infinitely nested radicals compute slightly differently. It is no surprise to see square roots along with trigonometric functions, given that we know identities such as  $tan(\pi/6) = \sqrt{3}$ . But it is completely unexpected to see multiple terms added to each other, with specific numbers showing up. It is mystifying how those infinite radicals, expressed so uniformly with radicals, addition and subtraction with one variable *a*, could have any relationship with an utter mess of radicals, 6s and 7s, multiplication and division and addition and subtraction, and an inverse tangent inside of a sine. The complexity of the expression on the one hand makes it more plausible, but on the other hand, makes us question how on earth anybody would think of something so convoluted.

Aside from being surprising, it is not clear what the use of such an expression would be. We are not dealing with a general statement about infinitely nested radicals, but with mathematical statements that appear to lack any generality whatsoever. This identity is only an evaluation of a few very specific examples of infinite radicals. Ramanujan does not even bother to use the symbol ±, which could produce a more general expression. To make matters worse, he gives no clue as to how he came up with this identity, or why it might be true. It simply hangs in front of us, and we don't know what to make of it. What was the point? Is this important in some way we don't see? Is it just meaningless algebraic manipulation? Why did he come up with While this is from his personal notebook and not any published results, and despite being at a considerably lower level of technical complexity than the rest of Ramanujan's work, this page captures some of his work's characteristic aspects. Ramanujan's work can strike us as astounding and wondrous; but its shock and unexpectedness can be confusing and daunting, and make his work seem of doubtful relevance or importance (especially because it is almost always unmotivated and unexplained). Ramanujan's notebooks, and many of his early published works, are filled with pages upon pages of results like this. He lists such results one after the other, without any hints of how he came up with them, or why he thinks that they are true. Ramanujan has the effect of a conjurer, and we wonder whether he was a charlatan practicing parlor-tricks, or perhaps a real *magus* (or more appropriately, a reality-twisting *yogi* or *rishi*).<sup>2</sup> There is even a perverse element to his wizardry; perhaps something so maddening could only be black magic<sup>23</sup>

I make these magical connections deliberately. For this is not a paper about Ramanujan's mathematics. Rather, it is about examining how various biographers have written about Ramanujan, how they have used and interpreted the evidence from his life, and how they created the portraits of Ramanujan we have today. I begin by quoting Ramanujan's mathematics to give

<sup>&</sup>lt;sup>2</sup> Ranganathan calls Ramanujan a *drashta*, divinely inspired seer, so this might be the most appropriate Sanskritic reference. S. R. Ranganathan, *Ramanujan: The Man and the Mathematician* (Bombay: Asia Publishing House, 1967), 22. Nandy, though, says that Ramanujan self-identified as a *yogi* within the available set of established Brahmin archetypes. Ashis Nandy, *Alternative Sciences: Creativity and Authenticity in Two Indian Scientists* (New Delhi: Allied Publishers Private Limited, 1980), 134.

<sup>&</sup>lt;sup>3</sup> Robert Kanigel, *The Man Who Knew Infinity: A Life of the Genius Ramanujan* (New York :Charles Scribner's Sons, 1991), 205-6. I take Kanigel's presentation, but for a more mathematically informed (though still dramatic) reaction, Berndt gives a tale left in notes of B. M. Wilson. George Pólya was visiting Cambridge in 1925, and borrowed from Hardy one of Ramanujan's notebooks. "A couple of days later, Pólya returned in a state of panic explaining that however long he kept them, he would have to keep attempting to verify the formulae therein and never again would have time to established another original result of his own." Srinivasa Ramanujan Aiyangar and Bruce C. Berndt, *Ramanujan's notebooks* Part I (New York: Springer-Verlag, 1985), 14. Cited hereafter as "Berndt, *Notebooks*," followed by the volume number (I through V), and the page number.

due share to the richness and unique character of the mathematical side of what Ramanujan has left us, and to give an idea of why mathematicians write such poetics about him. Also, most who have written about Ramanujan are not mathematicians, and hence rely heavily on what mathematicians say about Ramanujan. So the mathematical reaction to Ramanujan forms the basis for all other considerations. But why are non-mathematicians drawn to Ramanujan in the first place?

As the preeminent mathematician J. E. Littlewood wrote, "[Ramanujan's] definite contributions to mathematics, substantial and original as they are, must, I think, take second place in general interest to the romance of his life and mathematical career, his unusual psychology, and above all how great a mathematician he might have become in more fortunate circumstances."<sup>4</sup> The appeal of Ramanujan is in both the prestige given to him by mathematicians and his life story.

As the conventional story goes, Ramanujan was born (1887) in total poverty in a far-flung corner of the British Empire. He was a colonial subject at the height of imperialism at the turn of the 20<sup>th</sup> century. Without teachers, peers, institutional training or access to books, he came across an outdated and obscure textbook from which he taught himself mathematics. Then, he toiled away in obscurity, filling his personal notebooks with mathematics. He unknowingly rediscovered some of the greatest European mathematics of the last century and then went beyond it. Nobody around him could understand or appreciate his work, so he was stuck working in a clerkship to make enough money to survive. In the eyes of society, he was nothing but an insignificant clerk. But this lone Indian clerk wrote a letter to G. H. Hardy, the top

<sup>&</sup>lt;sup>4</sup> Littlewood, J.E. Mathematician's Miscellany (London: Methuen &. Co. Ltd., 1953), 85.

mathematician at Cambridge, asking Hardy to consider some of his enclosed mathematical results. The letter stunned Hardy, for in it he found theorems of a type he had never dreamed possible, theorems that delighted and baffled him. Hardy immediately lifted Ramanujan out of his poverty and obscurity, bringing him to Cambridge in 1913. While there, Ramanujan never gave up his Indian ways, maintaining strict vegetarianism and never conforming to his environment of British secular rationalism. In fact, he never saw any conflict between his Hindu faith and his advanced scientific work, and would say the goddess Namagiri would appear to him in his dreams and give him the most advanced mathematics. Ramanujan was the first Indian elected as a Fellow of the Royal Society, and the first Indian to become a Fellow of Trinity College, Cambridge. But the English weather and food took its toll on Ramanujan. After working with Hardy for six years, he became seriously ill. He returned to India in 1919, only to die less than a year later at the age of 32.<sup>5</sup>

It is the story of a great triumph and a terrible tragedy. It was a triumph of the impoverished over the privileged, of the colonial subject over the colonial masters, of faith and personal belief over the social conformity, and of the human mind over intellectual starvation and deprivation. Ramanujan overcame every obstacle imaginable to become one of the greatest mathematicians of all time. But while he ultimately triumphed, it was a terrible tragedy that he had to suffer and struggle for so long. And it was an ever greater tragedy when Ramanujan, exhausted from his struggle, died just as he achieved the fame and recognition he had deserved but been denied for so long.

<sup>&</sup>lt;sup>5</sup> This is drawn from an amalgam of accounts, which are discussed below.

It is a most wonderful story – but like any story this perfect and grand, it is a myth. It is crafted not with outright factual inaccuracies,<sup>6</sup> but with subtle issues of interpretation and presentation. Many small facts and details are liberally interpreted, and small twists that are individually harmless build upon one anther to craft a deceptively epic image of Ramanujan.

In my thesis, I argue that Ramanujan's various biographers have a specific set of underlying philosophical priorities (that they may or may not be aware of themselves); I explore what these are, and how they affect the interpretation of evidence and the overall presentation of Ramanujan. And I will argue that these assumptions and the resultant interpretations of Ramanujan are not the most desirable way to present him, that a different framework and approach to Ramanujan provides us with more interesting, useful and important insights.

In Chapter 1, I argue that the various accounts of Ramanujan, even when they explicitly disagree, all operate within the common framework of mathematical realism. This ontological position holds that mathematics is something independent of and prior to human conception, existing universally and eternally. The results of mathematical research are, on this view, discovered rather than invented or constructed. This framework, with its specific assumptions about the nature of mathematics, leads to an absence of critical interpretation of a great deal of evidence. I adopt a somewhat 'internalist' approach here, performing a sometimes narrow exegesis of Ramanujan's work and the comments about it. I do so in order to explore the realist accounts on their own terms, a task which often requires engaging with specific and technical claims. While sometimes this exegesis reveals that the realist-driven interpretations are wrong or

<sup>&</sup>lt;sup>6</sup> There are two outright but trivial factual inaccuracies: First, Ramanujan was not the first, but the second Indian elected to the Royal Society, though the previous election was in 1840. Second, Ramanujan's personal deity who would appear to him in dreams was Narashima, not Namagiri, the latter being the patron deity of his mother. These are mistakes that commonly appear in biographies, so I included these mistakes in my sample narrative.

extremely unlikely, the majority of cases are inconclusive and ambiguous. My task is not to disprove the realist interpretation outright, but only destabilize it, and show that another narrative is possible, one that better explains observed phenomenon.

In Chapter 2, I maintain the focus on the existing biographies, but expand my discussion to more contextual issues of Ramanujan's time that existing accounts neglect or downplay. I argue here for a different view of Ramanujan: not as an unprecedented exception whose life went from total obscurity to widespread fame, but as having been continuous with his community, with his peers, and within his own life. He was part of an emergent community of Indian scholars, including mathematicians, who were integrating into the British higher education system at the highest levels. This included those involved with higher education in India, and those who traveled to study in England. In the context of this revised narrative, Ramanujan's letter to Hardy was not such a drastic change, as Ramanujan's fame and support had been gradually rising for some time. I argue that while the existing accounts offer a sense of wonder and amazement, and that they try to heighten this sense by making Ramanujan as elevated and detached as possible, wonder precludes understanding. Ultimately these accounts are less satisfying than an account that helps us understand Ramanujan.

In the remainder of the introduction, I will discuss my methodology and sources. My methodology derives from particular trends within the History of Science, and is also informed by recent studies on the genre of scientific biography. In light of these studies, I will elaborate on the justification and aims of my project. Also, I will discuss the way in which I use mathematics, an often inaccessibly technical specialty. Finally, I will introduce the major primary and secondary sources we have relating to Ramanujan, and explain how I have read them.

#### Methodology

#### Realism and Mathematics in the History of Science

Realism<sup>7</sup> has been outmoded in academic history of science for at least forty years, and for good reason. Assumptions about scientific theories uniquely describing phenomenon, and having universal truth-value, break down under a historical view. Scientific theories of the past, now considered ridiculous and wrong, were once considered true. And previous, 'wrong' scientific theories were successful at describing the world and even predicting phenomenon – theories that were not just limiting cases of the current theory, but fully incompatible with the current theory. Such a historical perspective challenges the idea that current theories would have more integrity than theories of the past, thus undermining realist claims. Instead, studying how scientific knowledge is constructed has been enormously fruitful in understanding the development of science and explaining the inconsistencies in its history and development. Constructivism is not the only perspective in history of science; recently it has been critiqued for being realist about the existence of the 'social,'<sup>8</sup> although proposed alternatives seem only to transfer realist stability onto some other concept. But any such realism is still a far cry from the

<sup>&</sup>lt;sup>7</sup> By realism here I mean the assumptions that science describes the 'real world,' that scientific knowledge has a unique and universal truth-value, and that the objects of scientific theories are real. Realism in the philosophy of science is slightly different from realism in the philosophy of mathematics, mainly in a community split, where philosophers of mathematics are concerned with mathematics and use other science to aid the understanding of mathematics, and vice versa. I will elaborate on how I am defining and treating realism in Chapter I.

<sup>&</sup>lt;sup>8</sup> For example, Bruno Latour, *We Have Never Been Modern*, trans. Catherine Porter (Cambridge, MA: Harvard University Press, 1993).

crass, uncritical realism of the accounts we will encounter. Constructivism continues have great potential for subtle and nuanced insights, especially where it has never been applied.

Mathematics<sup>9</sup> has not generally benefited from constructivist approaches of history of science. There are three reasons for this. The first is that mathematical knowledge, more than any other scientific discipline, seldom stops using knowledge produced in its past.<sup>10</sup> A realist description of mathematics' millennia-long history, then, has few apparent inconsistencies. The second reason is the abstraction of mathematics. Mathematics seems as though it has no experiments, and no material culture – there is no "grubby, unplatonic equipment that lies such a long way from Lie algebras and state vectors,"<sup>11</sup> only those Lie algebras and state vectors themselves.<sup>12</sup> The third reason is that while mathematics does have inconsistencies, they are non-historical and there is an established disciplinary philosophy of mathematics addressing these.<sup>13</sup> The inconsistencies relate to the technical foundations of mathematics and not to its historical development, encompassing paradoxes such as those resulting from the concepts of infinities and sets. History of Science has little hope of being relevant in this discussion. It is about half as old

<sup>&</sup>lt;sup>9</sup> By which I mean mathematics from the nineteenth century until today, i.e. from the beginning of the continental tradition of analysis.

<sup>&</sup>lt;sup>10</sup> Hardy captured this perception when he wrote, "The history of mathematics shows conclusively that

mathematicians do not evacuate permanently ground which they have conquered once." G. H. Hardy, "Mathematical Proof," Rouse Ball Lecture in Cambridge University 1928, in *Collected Papers of G. H. Hardy, including joint papers with J. E. Littlewood and others* Vol. VII (Oxford: Claredon Press, 1979), 585.

<sup>&</sup>lt;sup>11</sup> Peter Galison, *Image and Logic: A Material Culture of Microphysics* (Chicago and London: University of Chicago Press, 1997), xvii.

<sup>&</sup>lt;sup>12</sup> Andrew Warwick challenges the perception that mathematics has no material culture, discussed below.

<sup>&</sup>lt;sup>13</sup> Properly speaking, the divide is between philosophy of science, and history/sociology of science. Then, while

philosophy of mathematics is distinct from philosophy of science, the two are closely related. There is also a divide within the history of science relating to mathematics. Until his recent passing, David Pingry headed the only History of Mathematics department in the world at Brown University (where he was the only professor). His work was related to technical reconstructions and comparisons, with no sociological treatment, and hence totally unrelated to disciplinary history of science. So there is history of mathematics as an independent but extremely obscure discipline. The divide I discuss is between the history of mathematics as a subset of history of science and the philosophy of mathematics as an independent field but related to the philosophy of science.

as the dominant positions in the philosophy of mathematics, which date to the early twentieth century. Because of this head start, the philosophy of mathematics addresses the inconsistencies of mathematics with experience and sophistication that it does not seem the newcomer history of science could match. The sophistication of philosophy of mathematics is especially apparent in its success in generating new mathematical work from the normative implications of its epistemological theories. When logicism proposed that all of mathematics could be reduced to logic, Russell and Whitehead attempted to do exactly this in technical terms in their monumental *Principia Mathematica.*<sup>14</sup> Hilbert's formalization program tried to reduce all of mathematics to symbolic manipulation, which opened mathematical ground that the incompleteness theorems of Gödel would soon fill.<sup>15</sup>

The unique abstraction of mathematics as compared to other scientific fields, and the uniqueness of its philosophical concerns, makes it seem as though mathematics might not even be a part of science. Within the history of science, Thomas Kuhn's highly influential essay "Mathematical vs. Experimental Traditions in the Development of Physical Science"<sup>16</sup> explores this divide by proposing two interrelated traditions in the history of science, the cluster of 'mathematical sciences,' and the empirical 'Baconian sciences.' Kuhn's essay importantly identifies that what has historically been considered part of a cluster of 'mathematical sciences' is not constant, for example with music dropping out of this cluster, and the study of motion joining the cluster. Kuhn affirms that math is a part of science, but he puts up an epistemological

<sup>&</sup>lt;sup>14</sup> "Introduction," *Philosophy of Mathematics: Selected readings, second edition*, Paul Benacerraf and Hilary Putnam, eds. (Cambridge: Cambridge University Press, 1983), 12-13.

<sup>&</sup>lt;sup>15</sup> Ibid, 7-8. Also, Georg Kreisel, "Hilbert's programme," *Philosophy of Mathematics: Selected readings, second edition*, 207-238.

<sup>&</sup>lt;sup>16</sup> Thomas S. Kuhn. "Mathematical vs. Experimental Traditions in the Development of Physical Science." *Journal of Interdisciplinary History* 7 (I) (Summer 1976): I-3I.

boundary within science defining a privileged place for mathematics by granting that the mathematical cluster of sciences operate differently from other sciences. But as subsequent sources have pointed out, Kuhn's essay is not empirically justifiable – indeed, Kuhn repeatedly stresses the tentative nature of his essay, and expresses confidence that an examination of the historical record will vindicate his view, a task that he encourages others to take up. J. A. Bennett's historical investigations are particularly influential, and he demonstrates that speculative mathematics was not distinguished conceptually or by practitioners from 'empirical' mixed-mathematical sciences, showing that Kuhn's epistemological boundary within science is not justifiable.<sup>17</sup> In other words, an essentializing view distinguishing mathematics from science is not historically justified. Hence, the disciplinary divide between philosophy of mathematics and history and philosophy of science has no basis in historical development.<sup>18</sup>

Then there is no essential reason that the critiques of realism in science would not apply to mathematics, and there is no reason such critiques should not also yield interesting insights about the historical development and epistemology of mathematics. This is especially critical because for many mathematical topics, historical and social treatments have been largely left to the popular press. Romanticizing biographies of Ramanujan, Erdös, or John Nash, or extremely linear descriptions of the development of specific concepts like zero or  $\pi$ ,<sup>19</sup> stand without even a critical counterpart, let alone a developed alternative in a constructivist history of mathematics.<sup>20</sup>

<sup>&</sup>lt;sup>17</sup> J. A. Bennett, "The Mechanics' Philosophy and the Mechanical Philosophy," *Hist. Sci.*, xxiv (1986): 1-28.

<sup>&</sup>lt;sup>18</sup> We might legitimately make an epistemological division if we do so normatively. But our project is descriptive.

<sup>&</sup>lt;sup>19</sup> Respectively, Robert Kanigel's *The Man Who Knew Infinity: A Life of the Genius Ramanujan*, Paul Hoffman's *The Man Who Loved Only Numbers: The Story of Paul Erdös and the Search for Mathematical Truth*, Sylvia Naser's *A Beautiful Mind*, Robert Kaplan's *The Nothing that Is: A Natural History of Zero*, and Petr Beckmann's *A History of*  $\pi$ . <sup>20</sup> While this is not to say that the history of science should take up biography writing, as the genre itself has inherent problems (see discussion in introduction); but lives overlap with a great deal of interesting extra-

Thomas Söderqvist, who studies the genre of scientific biography, writes, "Particular biographies – especially biographies of 'dead white males', like Galileo, Newton, Darwin, Huxley and Richard Feynman (and occasional 'dead white females' like Marie Curies and Dorothy Hodgkin) – have acquired an established, if not necessarily favoured, place in the awareness of most historians of science and students of science studies."<sup>21</sup> But when it comes to the history of mathematics, there is no similar awareness. Not for a lack of interestingly problematic biographical material – after all, Ramanujan perhaps is one of the only (if not the only) 'dead non-white male' frequently written about, which adds a whole new dimension to his case – but because of a lack of academic interest in mathematics among historians of science.

One of the few exceptions to this is Andrew Warwick's recent book, *Masters of Theory: Cambridge and the Rise of Mathematical Physics.* Warwick begins by noting, unlike studies of the material culture in experimental disciplines, "...when the tools of the trade are what have been described as '*readily transportable* mathematical techniques and abstract theoretical concepts,' the focus on craft skill, localism, and collective production is almost invariably abandoned."<sup>22</sup> Theoretical work is still treated as the work of isolated individuals. For example, Newton is still seen as having produced his work without relying on other individuals. Seldom do scholars ask, did Newton struggle with the material of his predecessors? Did he rely on 'lesser' scholars for insights? By not exploring questions of what resources Newton might have drawn upon,

biographical material, and discussions of this material is currently only available through these biographies.

<sup>&</sup>lt;sup>21</sup> Thomas Söderqvist, "Introduction," *The History of Poetics of Scientific Biography*, ed. Thomas Söderqvist, 1-15. (England: Ashgate, 2007), 4.

<sup>&</sup>lt;sup>22</sup> Andrew Warwick, *Masters of Theory: Cambridge and the Rise of Mathematical Physics* (Chicago: University of Chicago Press, 2003), 11-12.

historians perpetuate the assumption that he was isolated.<sup>23</sup> The same is so for Ramanujan, who is seen as having taught himself from a single textbook. While mathematical work might take place in isolation, neither Newton nor Ramanujan was socially isolated from a community of mathematicians. Instead of seeing British mathematician's comprehension and legitimization of Ramanujan's work as a miracle or evidence of the universality of mathematics, we can see that common knowledge, distributed along networks of shared competence, connected Ramanujan to these British mathematicians before they ever came into contact.

For comparing the pedagogies that produced a shared competence between Ramanujan and his British colleagues, I will rely on Warwick's discussions for the British context. While Warwick deals with mathematical physics and not pure mathematics, such a split within Cambridge did not become as important until 1909, when the 'Tripos' exam was reformed. Most of the major British mathematicians in Ramanujan's life had been trained in mathematics by preparing for the Tripos, a grueling, competitive, and immensely important nine-day exam that consumed the mathematical student's entire undergraduate career at Cambridge.

#### Scientific Biography

If a biography is supposed to give the reader a sense of the subject's life, then this is not a biography of Ramanujan. I do not give a picture of Ramanujan, nor do I strive to. What I look at above all else is how specific biographies construct portrayals of Ramanujan. In my treatment, if Ramanujan seems without agency, it is because he indeed lacked direct agency in the writing of these biographies, as he was not alive when they were written.

<sup>13</sup> 

<sup>&</sup>lt;sup>23</sup> Warwick 15-6.

Indirectly, telling the story of what was going on around Ramanujan, and telling the story of how others have portrayed him will give us some sense of who Ramanujan was. But this come nowhere close to being as evocative and rich as the image given by the biographical attempts to construct Ramanujan's life. In examining the possibility of doing history of science through biography, Mary Jo Nye comments, "While historians of science often use biography as a vehicle to analyze scientific processes and scientific culture, the most compelling scientific biographies are ones that portray the ambitions, passions, disappointments, and moral choices that characterize a scientist's life."<sup>24</sup>

I think a 'history of science' biography of Ramanujan is not appropriate for two reasons. First, unlike the figures that have been the subject of continued biographical attention within the history of science, Ramanujan is not associated with any conceptual shift. Biographical studies of 'dead white men' like Galileo or Darwin are inspired by a perception that they were responsible for an important conceptual shift, which biographies seek to understand. Ramanujan is not readily associated with any scientific idea, and neither was he responsible for any major institutional shift or change. In this sense, it is not appropriate to try and revise or "evaluate my subject's importance in some global or transhistorical sense."<sup>25</sup>

Second, the only materials we have authored by Ramanujan are 37 letters<sup>26</sup> and several notebooks filled exclusively with mathematics. Unlike scientists for which we have a rich archival

<sup>&</sup>lt;sup>24</sup> Mary Jo Nye, "Scientific Biography: History of Science by Another Means?" *Isis* 97 (2), (15 June 2006): 322-9.

<sup>&</sup>lt;sup>25</sup> Mary Terrall, "Biography as cultural history of science," *Isis* 97 (2), (16 June 2006): 308.

<sup>&</sup>lt;sup>26</sup> Making a count of the index of letters in Letters and Commentary 317-322. Even these are usually of an administrative nature and extremely short, with the only lengthy material being mathematics. These letters do tell a tremendous amount about how Ramanujan presented himself and communicated with particular individuals, but the secondhand comments we have about Ramanujan suggest that there is a much richer story about his self-identity than anything we could extract from his letters alone. While the comments together give a picture of Ramanujan's

record, we cannot reconstruct Ramanujan's daily activity.<sup>27</sup> While we can make some very limited comments about Ramanujan's self-identity as an actor's category,<sup>28</sup> a story about his 'self-fashioning' would be very meager. Thus, I do not try and examine how Ramanujan constructed an identity as a mathematician. I instead tell a rich story about how others constructed for Ramanujan an identity as a mathematician.

While my treatment is not a biography, there are two ways in which it is 'biographical.' First, at many points in Ramanujan's life I construct a biographical narrative. My narratives seldom give any insight into Ramanujan as an individual, as I use this narrative only as an alternative with which to compare the existing biographies. For example, my extended discussion in Chapter 1 about the extent of Ramanujan's rediscovery has little to do with Ramanujan. Its role is to provide an alternative to the narrative of rediscovery that other biographies take for granted in constructing their images of Ramanujan. Second, I do draw on the biographical technique of 'parallel lives.'<sup>29</sup> Especially in Chapter 2, I situate Ramanujan in a context of other individuals. Ramanujan was neither the only Indian doing mathematics in India, nor was he alone at Cambridge.

My project is, above all, historiographical. This is not about the cultural life of the scientist, though I make some comments about places where Ramanujan shows up in wider culture. Neither is it about mathematics as a collective enterprise, although I draw on the legacy

self-identity, critically taking each comment again becomes a study of how others have construed Ramanujan.

<sup>&</sup>lt;sup>27</sup> Bernadette Bensaude-Vincent, "Biographies as Mediators between Memory and History in Science," *The History and Poetics of Scientific Biography*, ed. Thomas Söderqvist (Burlington, VT: Ashgate, 2007), 178.

<sup>&</sup>lt;sup>28</sup> David Aubin and Charlotte Bigg, "Neither Genius nor Context Incarnate: Normal Lockyer, Jules Janssen and the Astrophysical Self," *The History and Poetics of Scientific Biography*, ed. Thomas Söderqvist (Burlington, VT: Ashgate, 2007), 62.

<sup>&</sup>lt;sup>29</sup> Terrall 308; Auburn and Bigg 55.

of approaching science as a collective enterprise. I focus on developed individual attempts at presenting a biography of Ramanujan that involve comprehensive and original research. My project is about closely examining the historiography of these accounts. This yields some fascinating results because, as we will see, a wide variety of authors have written about Ramanujan.

#### Use of Mathematics

Recall the mathematical example given at the beginning, and specifically, its unexpectedness. Without any evidence of a process, if we and try and fail to reconstruct some possible line of reasoning, we might begin to feel as though this came from a leap of intuition. Many of the biographers of Ramanujan are non-mathematicians, and must either take mathematicians' poetic comments about Ramanujan's mathematics at face value, or avoid talking about Ramanujan's mathematics. Middle ground is impossible without dealing with the mathematics. Not talking about this central part of Ramanujan's life obviously would be a gross omission, but accepting mathematician's characterization carries the grave risk of making impossible critical assessment of this part of Ramanujan's life.<sup>30</sup> By and large, the latter has happened in biographies of Ramanujan. As a result, his mathematics has become locked inside a black box of intuition and incomprehensibility, from which emanates Ramanujan's mystique and aura of genius.

<sup>&</sup>lt;sup>30</sup> This is exactly what happens with Ashis Nandy's critical treatment. All his critical work happens on top of a foundation of taking mathematicians' comments about Ramanujan's mathematics at face value. See my discussion below in Chapter 1.

This black box also enables mystical interpretations of Ramanujan. A story that has circulated a great deal is that Ramanujan had a dream where he saw a red screen formed of flowing blood, on which a hand traced out elliptic integrals. As soon as Ramanujan awoke he wrote them down. Drops of blood are a symbol of the god Narashima, a lion-faced incarnation of Vishnu, and Ramanujan claimed that after seeing such drops, "scrolls containing the most complicated mathematics used to unfold before him."<sup>31</sup> When we cannot explain how Ramanujan came up with his results, interpretations like this one gain traction.

But it is extremely unlikely Ramanujan's mathematics came inexplicably from dreams. As the leading Ramanujan scholar Bruce C. Berndt writes, "...for most of his work, we have no idea how Ramanujan made his discoveries... [Ramanujan's widow S. Janaki] remarked that her husband was always fearful that English mathematicians would steal his mathematical secrets while he was in England. It seems that not only did English mathematicians not steal his secrets, but generations of mathematicians since then have not discovered his secrets either."<sup>32</sup> In other words, Ramanujan did have secrets, he did have methods, but he kept them private for fear of theft. As we shall see in Chapter 2, as soon as Ramanujan had the opportunity to express his results in an officially sanctioned record (which would be a guarantee of priority), he did reveal specific mathematical tools, techniques and approaches that he had used to generate previously expressed results.

Bruce Berndt has made his life's work a study of Ramanujan's mathematics. For much of the work in which Ramanujan provides only lists of results, Berndt has attempted to reconstruct

<sup>&</sup>lt;sup>31</sup> Rajagopalan's reminiscences, quoted in Ranganathan 87. I elaborate on the circumstances of this claim below under the discussion of sources.

<sup>&</sup>lt;sup>32</sup> Berndt, Notebooks I, 9.

Ramanujan's thought processes, speculating about heuristic or formal arguments that Ramanujan might have used to justify his results to himself absent of a proof. Although Berndt feels he has not 'discovered Ramanujan's secrets,' his reconstructions go a long way towards doing away with the opacity on which mystical explanations thrive.

While the facsimile page pictured earlier is taken out of the context, an examination of the preceding and following notebook pages does not provide any immediate explanation for the results on this page. In his study of this chapter of Ramanujan's notebooks, Berndt finds that the nested radicals are a result of the system of equations,  $\{x^2 = y + a, y^2 = z + a, z^2 = x + a\}$ . Solving for *x* by substituting in for *y* and *z*, and then substituting in again for *x*, we can express *x* as an infinitely nested radical. Conversely, if instead of substituting back in for *x*, we simplify instead, we get an eighth-degree polynomial. This polynomial factors into a quadratic and two cubic polynomials. As the roots of these polynomials are solutions for *x*, the roots will be equal to the infinitely nested radical. Solving for the roots yield the identity that Ramanujan wrote down; the trigonometric functions come from an algorithm that uses a particular trigonometric identity to solve cubic equations. Variations in plusses and minuses, of course, give the variations pictured in this page from Ramanujan's notebook.<sup>33</sup>

This is only a brief description of the method Ramanujan probably used, for a complete discussion I refer the reader to Berndt's analysis. The point relevant for this essay is not exactly how Ramanujan arrived at a result, but that Ramanujan did indeed arrive at this result and it did not come from a supernatural source. My subsequent use of mathematics will be similar in scope

<sup>&</sup>lt;sup>33</sup> Berndt, *Notebooks* IV, 10-17.

to this case. I use mathematics insofar as an examination of Ramanujan's mathematics is necessary to critique portrayals of Ramanujan that black-box his mathematics.

What I will not do with Ramanujan's mathematics is extract from it conclusions about his personality or sense of self-identity. Ramanujan's mathematics does tell us about his worldview, but it tells us about it in a formal language. If we try and perform an exegesis and translate this formal language into vernacular terms, we end up with trivial and banal statements. For example, his preference for infinite series, and his disregard of conditions of convergence, suggests a concern with the transcendental over immediate experience, but the worldview expressed in such a statement is nothing compared to the insight into Ramanujan's view of the world given by actually experiencing his mathematics. Trying to translate from this formal language into our vernacular yields as meaningless results as if this paragraph were translated into mathematics; while both are connected in that they are anthropomorphic expressions, the type of content expressed in one is outside of the expressive ability of the other. At the start, in order to convey any of the sense of Ramanujan's mathematics, I have had to include an untranslated 'quote' of the formal language (the facsimile page), for there was no way other to convey any measure of actual understanding of Ramanujan's mathematics.

Ramanujan's mathematics is autobiographical in the sense that studying it informs us about Ramanujan.<sup>34</sup> But such study can only take place in the terms of the formal language.

<sup>&</sup>lt;sup>34</sup> Here I recall Nietzsche's comment that "every great philosophy so far has been... the personal confession of its author and a kind of involuntary and unconscious memoir..." *Beyond Good and Evil* §6, trans. Walter Kaufmann. Even when considering mathematics, I would contend that content is not separable, distinct or even distinguishable from author. Mathematical content is not conveyable abstractly, it is always conveyed through the personal interpretation of a mathematician. Unless a mathematician is copying some proof verbatim, there will be subtle differences in the way in which steps are expressed in which elements of an identity and worldview will be expressed. Perhaps with Ramanujan this is less contentious than for mathematicians in general, because Ramanujan's

Research into Ramanujan's mathematics gives us insight into the person, but such insight can be conveyed in vernacular only to others who have already experienced the content of the formal language. Even then, such discussions will usually be heavy with 'quotes' in the formal language.<sup>35</sup>

I do not think that this untranslatability implies or justifies separation, though. We will not understand Ramanujan without his mathematics, nor will we understand him with only his mathematics. But as this study is not about understanding Ramanujan, I will not delve in any depth into Ramanujan's mathematics. Nor will I try to compensate for this by drawing psychological conclusions from Ramanujan's mathematics.<sup>36</sup> What I do is analyze the way in which existing biographies derive or accept ontological and epistemological conclusions from Ramanujan's mathematics.

### Historiography

#### Primary Sources

As mentioned above, we have very little authored by Ramanujan himself: about three dozen letters, and notebooks filled exclusively with mathematics. Bruce C. Berndt and Robert A.

mathematics has such a unique and distinct character and style.

<sup>&</sup>lt;sup>35</sup> For an excellent example, see Freeman J. Dyson, "A Walk through Ramanujan's Garden," *Ramanujan: Essays and Surveys*, Bruce C. Berndt and Robert A. Rankin, eds (US: American Mathematical Society, 2001), 261-276.

<sup>&</sup>lt;sup>36</sup> Here, Alan Sokal's critique of attempts at 'boundary transgression' is particularly relevant. Alan D. Sokal and J. Bricmont, *Fashionable Nonsense: Postmodern Intellectuals' Abuse of Science* (New York: Picador USA, 1998).

This untranslatability (note: *not* incommensurability) agrees with Sokal in that we cannot drawn humanistic implications from scientific theories (and that social studies of science do not have the capability to change scientific theories). But, as expressed in the coexistence of the mathematical and non-mathematical in Ramanujan, I somewhat paradoxically say that despite this untranslatability these sides cannot be separated for any genuine understanding. My position is similar to that of Warwick: attempts at social explanations of cognitive judgments too strongly imply that mathematics can be understood by reference to extrascientific social factors, and too weakly examine the forms of sociability unique to and inseparable from technical practice. Warwick 44-45.

Rankin have collected all the epistolary sources relating to Ramanujan – including after his death – in the invaluable volume *Ramanujan: Letters and Commentary*.<sup>37</sup> This includes letters collected from private sources, material from the Trinity College archives, some translations of material in Tamil (Ramanujan's native language) including a 'family record' written by one of Ramanujan's otherwise obscure two younger brothers. Berndt and Rankin not only collect all available sources, but also provide commentary for each letter. When possible, the commentary includes the circumstances of its composition, the identities of the author, addressee, or people mentioned therein, and corrections of any mistaken information in the letters. There is also extensive technical commentary on the mathematical material found in Ramanujan and Hardy's letters to each other.

The other major primary source is Ramanujan's notebooks, which are exclusively mathematical in content. Except for some material on magic squares that dates from earlier, the material in the notebooks was composed between 1903 and 1914, stretching from Ramanujan's last year of high school until his departure for England. We have a total three notebooks.<sup>38</sup> The first notebook is organized into 16 chapters with a total of 134 pages, plus about 80 pages of disorganized material at the end. Initially Ramanujan only wrote on one side of the paper, but as the notebook went on, he began using the reverse pages for scratch work, and eventually began

<sup>&</sup>lt;sup>37</sup> Bruce C. Berndt and Robert A. Rankin, *Ramanujan: Letters and Commentary* (US: American Mathematical Society, 1995). Cited hereafter as *Letters and Commentary*.

<sup>&</sup>lt;sup>38</sup> There is also a 'Lost Notebook,' which (as admitted by those who dubbed it as such) is properly neither lost nor a notebook. It is a loose collection of papers, and was found among a collection of papers of the late professor G. N. Watson (one of Ramanujan's collaborators who undertook the first systematic effort to edit Ramanujan's notebooks) upon his death. Based on the circumstances in which it was found and its content, this was almost certainly written in the last year of Ramanujan's life, when he had returned to India. George E. Andrews and Bruce C. Berndt, *Ramanujan's Lost Notebook*, Part I (New York: Springer, 2005), 1.

This, along with other unpublished material that dates from after Ramanujan's departure for England, is not relevant for the present study and we will not draw upon it.

recording results there as well. The second notebook has 21 chapters with a total of 252 pages, followed by about 100 pages of disorganized material. The third notebook is 33 pages of unorganized material.<sup>39</sup>

Ramanujan kept his notebooks as a record of results he would work out on a chalk slate; the prohibitive cost of paper prevented him from using paper for scrap work (which might partially account for Ramanujan's style of presentation in the notebooks). These notebooks found their way into the archives of the University of Madras. In an effort to make them available, in 1957 the Tata Institute for Fundamental Research published a facsimile copy in two volumes. These volumes are without commentary, and contain only page-by-page reproductions of Ramanujan's notebook (including preserving when Ramanujan wrote on one or both sides of a page). We will draw upon these notebooks for historical information about what Ramanujan was working on before his departure to England, as well as the form in which he expressed certain propositions. The latter detail is often crucial for reconstructing the extent of Ramanujan's rediscovery.

There are also collections of oral records, presented mainly in the biographies of Ramanujan by S. R. Ranganathan and P. K. Srinivasan. The work of the latter is largely unavailable, but Srinivasan provided most of its content as well as some additional, unpublished material to Berndt and Rankin for inclusion in *Letters and Commentary*. Ranganathan devotes a chapter to quoting reminiscences of people who knew Ramanujan within a larger biographical work. This biography is one of the most idiosyncratic, and as we will see, the oral records

<sup>&</sup>lt;sup>39</sup> Berndt, Notebooks I, 5-6.

contained therein are often not reliable enough to be taken at face value. Hence, we will consider this along with secondary sources.

A companion volume to *Letters and Commentary*, also edited by Berndt and Rankin, is *Ramanujan: Essays and Surveys*. This contains the most significant essays about Ramanujan, but also contains some primary source material. There is a record of Ramanujan's exam scores, a translation of another family record with contributions by both of Ramanujan's two younger brothers, a transcript of an interview with Ramanujan's widow S. Jannaki, and other miscellaneous material.

#### Secondary Sources

These are 'secondary sources' only in the sense that they are secondary descriptions of Ramanujan; much of what I do is treat these as primary sources. Most of the information about Ramanujan in preserved only in these secondary sources, so treating them as primary sources and performing an exegesis on them will often be the most reliable method of gathering information related to Ramanujan.

The earliest (and for some time only) biography of Ramanujan was written by two of his advocates and benefactors in India, P. V. Seshu Aiyar and Ramachandra Rao.<sup>40</sup> This biography was composed for inclusion in the volume of Ramanujan's *Collected Papers*,<sup>41</sup> published in 1927 as a commemorative volume. <sup>42</sup> I will not consider Aiyar and Rao's biography in itself, as it is

<sup>&</sup>lt;sup>40</sup> Ramanujan's relationship with these two figures will be explained further in Chapter 1.

<sup>&</sup>lt;sup>41</sup> Srinivasa Ramanujan Aiyangar, G. H. Hardy, P. V. Seshu Aiyar, and B. M. Wilson, *Collected Papers of Srinivasa Ramanujan* (Cambridge: The University Press, 1927).

<sup>&</sup>lt;sup>42</sup> Ranganathan, in his 1967 biography (pg 15-17), claims that in 1923 he made an initial draft for the biography that appeared in the *Collected Papers*. While his name is not mentioned anywhere in the *Collected Papers*, at the time he

very short. But some widely quoted anecdotes about Ramanujan originate in this biography, and when we examine sources containing those anecdotes, we will revisit Aiyar and Rao's biography.

While this was the first biography, there were earlier obituaries. The first obituary was in 1920, by G. H. Hardy in an issue of *Nature*.<sup>43</sup> This obituary was reprinted, along with two original obituaries by Aiyar and Rao, in a 1920 issue of the *Journal of the Indian Mathematical Society*.<sup>44</sup> These were followed by an obituary by E. H. Neville in a 1921 issue of *Nature*.<sup>45</sup> Hardy wrote another obituary that appeared in the *Proceedings of the London Mathematical Society*,<sup>46</sup> and was reprinted both in the *Proceedings of the Royal Society*<sup>47</sup> and in the *Collected Papers*. The content of Aiyar's obituary is very similar to the later biography in the *Collected Papers*, but Rao's obituary contains some comments not found elsewhere, and I will quote it as necessary.

The first in the set of biographies I focus on is that of G. H. Hardy. While not a biography proper, as Hardy relies on Aiyar and Rao's account for biographical details about Ramanujan's life, Hardy provides an extremely specific and rich interpretation of Ramanujan. This interpretation was delivered in a series of lectures Hardy gave in 1936, and subsequently published as *Ramanujan: Twelve Lectures on Subjects Suggested by his Life and Work*. Hardy's account has been very influential for two reasons; first, he has the authority of not just a professional mathematician who can understand Ramanujan's work, but the authority of

was a student, and it might have been a case of the professor (specifically, Seshu Aiyar) passing labor off to the student without credit. The relevant question is how much Ranganathan acted in a clerical function as a scribe or compiler, versus how much of the biography was his original work. This is not an important issue, but I include this discussion to note that the account of Aiyar and Rao and that of Ranganathan might not be independent of each other.

<sup>&</sup>lt;sup>43</sup> Vol. 2642, no. 105.

<sup>&</sup>lt;sup>44</sup> Vol. XII, no. 3.

<sup>&</sup>lt;sup>45</sup> Vol. 2673, no. 106.

<sup>&</sup>lt;sup>46</sup> Vol. XIX, no. 3, pp. xl-lviii.

<sup>&</sup>lt;sup>47</sup> Series A, Vol. XCIX, pp. xiii-xxix.

mathematician who worked most closely with Ramanujan and hence best poised to be an expert on Ramanujan's work. Second, as Hardy elsewhere writes,<sup>48</sup> if he had not been a mathematician he would have been a journalist; he took great pride in writing, and he was extremely eloquent author. Hardy provides both authority and eloquence, and becomes the ideal source to quote. For Hardy we also have a personal memoir of sorts, *A Mathematician's Apology*, written in 1940 shortly after the lectures on Ramanujan. Although mentioning Ramanujan only twice, this work gives great insight into how Hardy thought about mathematics. We can draw on this, as well as numerous miscellaneous writings of Hardy, to inform our reading of his interpretation of Ramanujan.

The next account, chronologically, is that of S. R. Ranganathan in 1967. According to his own account, Ranganathan was a junior member of the staff at the Department of Mathematics at Presidency College at the time of Ramanujan's death.<sup>49</sup> He explains that, as time went on, he noticed increasing numbers of Indian mathematics students were totally ignorant of Ramanujan, or had only heard of the name and little more. This biography is an attempt to spread knowledge of Ramanujan.

Ranganathan is a perfect example of a biography serving a 'programmatic' function.<sup>50</sup> He states his intention to provide a biography to publicize Ramanujan,<sup>51</sup> but does not say what he hopes to accomplish by this. It becomes increasingly clear through the course of the work.

<sup>&</sup>lt;sup>48</sup> G. H. Hardy, *A mathematician's apology* (Cambridge: The University Press, 1940), 150. Cited hereafter as Hardy, *Apology*.

<sup>&</sup>lt;sup>49</sup> Ranganathan 9. Presidency College is presented in Chapter 1.

<sup>&</sup>lt;sup>50</sup> Signe Lindskov Hansen, "The Programmatic Function of Biography: Readings of Nineteenth- and Twentieth-Century Biographies of Niel Stensen (Steno)," *The History of Poetics of Scientific Biography*, ed. Thomas Söderqvist, 135.

<sup>&</sup>lt;sup>51</sup> Ranganathan 10.

Ranganathan seeks to present Ramanujan as the model of an ideal person, to inspire students and especially student of mathematicians. Ranganathan's account has moments of pure hagiography, in the most literal sense: he compares Ramanujan to Rama,<sup>52</sup> the hero of the great <sup>54</sup> but Indian epic the Ramayana, <sup>53</sup> who is not only the traditional image of the ideal man nothing less than God incarnate.<sup>55</sup> Even considered as poetic devices and classical allusions, there was still a choice of which classic figure to allude to. Ranganathan does not make these classical allusions in a humanistic sense; at least some of it he means quite literally. He speaks of Ramanujan being an inspired seer, and takes quite seriously the stories of Ramanujan being inspired by a divine entity in dreams. He includes a story where he and a colleague contacted the deceased Ramanujan with a Ouija board, and quite seriously claims that Ramanujan communicated that he had stopped working on mathematics in his afterlife and had devoted himself to religious pursuits.<sup>56</sup> Ranganathan recognizes a "common allergy to trans-rational phenomenon," and his argument is that it is not possible to understand Ramanujan in anything but 'trans-rational' terms. This is precisely the proposition we will investigate by looking at Ranganathan's use of sources.

Ranganathan includes a chapter of reminiscences he has collected from people who had known Ramanujan, which has been treated by all subsequent accounts as a major primary source. Considering the rest of Ranganathan's work, we must consider the possibility that these

<sup>&</sup>lt;sup>52</sup> "[From the Ramayana:] 'When one sees Rama, who will think of any of his organs except his eyes?' ... Also in the case of Ramanujan." Ranganathan 92.

<sup>&</sup>lt;sup>53</sup> There is an etymological connection between the two names: the Sanskrit form of Ramanujan's name, "Ramanujaha," means "little brother of Rama." *Letters and Commentary* 101. However, names derived from and including 'Rama' are extremely common, so this connection is of no significance.

<sup>&</sup>lt;sup>54</sup> The ideal woman is his wife, Sita.

<sup>&</sup>lt;sup>55</sup> Swami Venkatesananda, *The Concise Ramayana of Valmiki* (Albany, NY: State University of New York Press, 1988),

<sup>&</sup>lt;sup>56</sup> Ranganathan 16.

comments are presented selectively or censored by Ranganathan. This is in addition to possible self-censorship of the interviewed individuals, or possible selective memory over the gap of four decades erasing all but the most glorious aspects of this enshrined Indian national hero. The comments never contradict each other or Ranganathan, and present a uniform story. They all remark on Ramanujan's virtues, including his religious devotion, and some talk about occult phenomenon relating to him.

In assessing the trustworthiness of these reminiscences, we take the example of T. K. Rajagopalan, who claims to have known Ramanujan as a boy. Rajagopalan describes sheltering Ramanujan for a night, and Ramanujan reporting in the morning having dreamt of elliptic integrals traced out on a screen of flowing blood. Rajagopalan adds that this account of Ramanujan is "an extract from my book *Hidden Treasures of Yoga*." Rajagopalan slips in an advertisement for his book! The title of the book suggests a religious or devotional work, not one dedicated to historical rigor and accuracy. In addition, Rajagopalan published his own biography of Ramanujan in 1988, which is so cute in its retellings of Ramanujan anecdotes that I could not bring myself to seriously consider it. This case shows us that the volume of reminiscences quoted here are not trustworthy just because Ranganathan does not speak them, that each person likely has their own agenda and preoccupation.

Ranganathan's account and the reminiscences he collects is suspect, but for aspects of Ramanujan's daily life it is only source we have. Ranganathan was the only one who collected reminiscences of anybody he could find that had known Ramanujan. I do not reject Ranganathan's account outright, but try to examine individual reminiscences on a case-by-case basis. The next account, from 1980, is unusual compared to the other accounts. This is the psychoanalytic study of Ramanujan and Hardy by the postcolonial scholar, Ashis Nandy. I include this study though it is more analytical with only a bare biographical narrative for two reasons. First, Nandy conducted original research, and cites interviews and discussion with specific figures. For example, Nandy was the first to discover and publish the names of two Cambridge mathematicians that Ramanujan had contacted prior to Hardy but who had either ignored or rebuffed him, a story that had been covered up to save the two from embarrassment.<sup>57</sup> Second, Nandy is, as far as I have found, the only critical treatment of Ramanujan. I found analyzing this critical attempt fascinating, especially in comparing Nandy's account to the accounts of Ranganathan and Hardy that he critiques. Nandy's use of evidence is unlike any of the other sources, but his critical project fascinatingly falls into the same traps of assumption as the other accounts.

Last and most importantly, there is the 1991 popular biography of Ramanujan, *The Man Who Knew Infinity: A Life of the Genius Ramanujan*. This highly acclaimed work probably did – and continues to do – more to bring Ramanujan into the popular consciousness than anything else. Ranganathan and Nandy were published in India; while I do not know about their accessibility there, they are certainly very obscure outside of India. The accounts of Hardy, or Aiyar and Rao, are part of mathematical works and thus not in normal circulation. Only Kanigel's account is readily accessible.

<sup>&</sup>lt;sup>57</sup> Once Ramanujan had become a celebrity, Hardy, Littlewood and others who knew of this spared them from embarrassment. C. P. Snow had revealed the existence of these two figures in his 1967 preface to Hardy's *Apology*, as they had both passed away, but does not give their names. In an interview, Littlewood, also citing the two mathematician's passing as justification for revealing the secret, tells Nandy who the two were.

Kanigel's tells a masterpiece of a story, and in the process constructs a Ramanujan that both depends on and builds on the Ramanujans of the previous accounts. An impressive amount of scholarship and research has gone into Kanigel's work, a great deal of it original. Kanigel traveled to south India to visit the locations (and when possible, interview the people) associated with Ramanujan, he looked at material in the Trinity College archives (subsequently published in Letters and Commentary), and he carefully cites in endnotes the source of the assertions he makes. The research is admirable, but Kanigel is, in the end, telling a story. He puts words in mouths or thoughts in the heads of the characters, takes liberties in making imaginary reconstructions of micro-narratives, and makes dramatic and emotionally charged transitions. Reflecting Kanigel's priority of telling a good story, the Ramanujan that emerges is a tragic figure. Tragedy need not be confined to literary genres, and in fact is an ideal form in which to present genius.<sup>58</sup> The kind of simple hagiography, where the subject is perfect, is boring. In tragedy, the trope of unavoidable, inescapable loss is a contrasting element that emphasizes and more powerfully evokes a sense of the epic, and the uncontrollability of it exonerates the subject from any blame or responsibility that might diminish his or her perfection.

Kanigel writes in his prologue, "[this] is not a story that concludes, *Genius will out*—though Ramanujan's in the main, did... this is also a story about social and educational *systems*, and about how they matter, and how they can sometimes nurture talent and sometimes crush it."<sup>59</sup> I think here Kanigel is recognizing how a biographer's relationship with a subject

 <sup>&</sup>lt;sup>58</sup> Christopher A. J. Chilvers, "The Tragedy of Comrade Hessen: Biography as Historical Discourse," *The History and Poetics of Scientific Biography*, ed. Thomas Söderqvist. 105-7.
 <sup>59</sup> Kanigel 3.

risks falling into hagiography.<sup>60</sup> While he succeeds in this, what he ends up with is not a more nuanced picture of social and pedagogical factors influence on individuals, but a quintessential tragedy. Kanigel provides no serious analysis of the institutions in themselves, only in how they oppressed the helpless hero Ramanujan. The only sense in which this is not a perfect tragedy is the lack of a tragic flaw in the hero (though one that is unrelated with his or her eventual ruin). While Kanigel provides a small discussion of how Ramanujan's pride contained arrogance and elitism, this attempt perhaps at providing a more 'balanced' view of Ramanujan is unconvincing. The discussion is small, not well integrated into the rest of the narrative, and far outweighed by the impression of the heroic and perfect Ramanujan of the rest of the story.

One of the few, and possibly only, reward system in place for science is recognition. Scientific biography is a prime way to carry out this recognition.<sup>61</sup> Kanigel's biography is very much a project of recognizing Ramanujan, of rewarding him for his perseverance and achievement (or rather, rewarding his historical legacy, which amounts to rewarding people who resemble him). Thomas Hankins points out a curious paradox inherent in scientific biography as a reward system. There is a distinction between a "career" and a "life;" a career is the basis on which awards are given, but the biography is about the life.<sup>62</sup> But a life is made up of religion, education, politics, social status, health, race and ethnicity, gender, marriage, and sexual preference, exactly the things that a career should be rewarded independently of, not to mention

<sup>&</sup>lt;sup>60</sup> Terrall 308.

<sup>&</sup>lt;sup>61</sup> Thomas L. Hankins, "Biography and the Reward System in Science," *The History and Poetics of Scientific Biography*, ed. Thomas Söderqvist, 93.

<sup>&</sup>lt;sup>62</sup> Note Kanigel's carefully worded subtitle: "A life of the genius Ramanujan." The indefinite article humbly presents this not as a definitive biography but only one possible presentation, and Kanigel's use of "life" here implicitly provides a disclaimer that this aspires only to tell about Ramanujan's life and not about his career.

that tying the production of science to personal, social and cultural factors is a scientific taboo.<sup>63</sup> Then, how can a biography tell the story of a scientist and simultaneously praise him? Kanigel's popular biography provides a resolution: Kanigel masterfully describes every detail of the context around Ramanujan, and yet manages to present a Ramanujan who was wholly untouched and unaffected by it in some fundamental way. This contrast between what Kanigel presents and how he interprets it makes for a fascinating exegesis.

One of my major sources, and the only one I do not treat critically, is Bruce Berndt's editions of Ramanujan's notebooks. To a certain extent I will do exactly what I criticize other accounts for doing, relying on the assertions of mathematicians, when I trust Berndt's historical analysis and characterizations of Ramanujan's mathematics. I do this first because Berndt is a more trustworthy source than Hardy; not only do Berndt comments come across as more measured and reasonable than Hardy's dramatic and poetic declarations, but also, by having made the study of Ramanujan's mathematics his life's work, Berndt actually has greater familiarity with Ramanujan's work than Hardy had. Second, unlike the other sources accept wholesale the mathematicians' analysis of Ramanujan, I delve fully into Berndt's editions of Ramanujan, and do not just take his general comments and characterizations. At certain, very specific points, I disagree with him. While there are overtones of realism in his presentation, I have found Berndt's editions as much 'without bias' as imaginable, partly because so much of this work is expressed in a formal language in which our vernacular term 'bias' does not have a meaningful equivalent.

I do, however, stress that there is an important gap in presentation between Ramanujan

<sup>&</sup>lt;sup>63</sup> Hankins 308. The list of factors is taken verbatim from Hankins.

and Berndt. For example, entry 1 of Chapter 3 in Berndt (*Notebooks* I, 45) reads (where summations are from k, j, n = 0 to  $\infty$ ):

Let f(z) be analytic on  $|z| < R_1$ , where  $R_1 > 1$ , and let  $g(z) = \sum Q_k z^k$  be analytic on  $|z| < R_2$ , where  $R_2 > 0$ . Define  $P_k$ ,  $0 \le k < \infty$ , by  $\sum P_k z^k = e^z g(z)$ , where  $|z| < R_2$ . Suppose that  $\sum Q_j \sum f^{(j+k)}(0)/k!$  converges and that this repeated summation may be replaced by a summation across diagonals, i.e., j + k = n,  $0 \le n < \infty$ . Then:

$$\sum P_n f^{(n)}(0) = \sum Q_n f^{(n)}(1).$$

Whereas, Ramanujan's original notebook (Tata Institute Vol. II, 23) simply reads:

$$\begin{split} &If P_0 + P_1 x + P_2 x^2 + P_3 x^3 + \&c = e^x (Q_0 + Q_1 x + Q_2 x^2 + \&c) \\ &Then \ P_0 f(0) + P_1 f''(0) + P_2 f'''(0) + P_3 f'''(0) + P_4 f^{IV}(0) + \&c \\ &= Q_0 f(1) + Q_1 f''(1) + Q_2 f'''(1) + Q_3 f'''(1) + Q_4 f^{IV}(1) + \&c. \end{split}$$

Note how Berndt has reformulated the statement in terms of analytic functions, while Ramanujan introduces the function f without any context. Berndt's edits, while putting Ramanujan's incomprehensible notebooks into forms usable by the mathematical community, obscure the original form in which Ramanujan presented his work. Hence, when I use to something cited in Berndt, I cross-check it with the Tata Institute Facsimile volumes to make sure that Ramanujan's original notation or formulation does not tell a different story than Berndt's translation into standard terms.

There are numerous other sources that I have elected not to consider, but which I will make mention of. There are numerous small treatments of Ramanujan in India, some by the Indian government and some by private individuals, often in regional languages. I have found what appear to be accounts of Ramanujan in Hindi, Gujrati, Marathi, Kannada, Telegu, and Tamil. There is a rich story to be told about Ramanujan's portrayal in India, but that is not what I have engaged with. The only other noteworthy nonfiction treatment of Ramanujan I know of is a documentary co-produced by the BBC and WGBH in 1988, titled *Letters from an Indian Clerk* in Britain and *The Man Who Loved Numbers* (not to be confused with Paul Hoffman's 1999 biography of Paul Erdös, *The Man Who Loved Only Numbers*) in the US.

I close by mentioning Ramanujan's cultural life. Perhaps Ramanujan's most prominent cultural appearance is as the subject of a dialogue in the 1997 movie *Good Will Hunting*, a wonderful example of realism at its most crass:

LAMBEAU: "This boy [Will Hunting] is incredible. I've never seen anything like him."

SEAN: "What makes him so incredible, Gerry?"

LAMBEAU: "Ever heard of Ramanujan?"

SEAN: "Yeah, right." [Pause] "... no."

LAMBEAU: It's a man. He lived over a hundred years ago. He was Indian. Dots, not..."

SEAN: [laughing] "...feathers, yeah."

LAMBEAU: "And he lived in this tiny hut somewhere in India. He had no formal education. He had no access to any scientific work. But he came across this old math text. And from this *simple text* he was able to extrapolate theories that had baffled mathematicians for years."

SEAN: "Yes! Continued Fractions? He wrote a, a-"

LAMBEAU: "Well, he mailed it to Hardy in Cambridge. And Hardy immediately recognized the brilliance of his work, and brought him over to England. And they worked together for years, creating some of the most exciting math theory ever done. This Ramanujan, his genius was *unparalleled*, Sean. —This boy is just like that."<sup>64</sup>

These cultural depictions move beyond the realm of secondary sources and into the realm of fiction, which is beyond my consideration. But it is interesting to note here how Ramanujan is

<sup>&</sup>lt;sup>64</sup> Transcribed from the film.

mobilized, what he is used to symbolize; here, the isolated genius. It is very much the same as in the sources I analyze. While I do not analyze cultural life, my study provides some interesting insights about the concerns that lead people to care about Ramanujan, and how those concerns play into the act of interpreting history. Ramanujan often acts as a powerful symbol, and we now set out to see what this symbol is, and explore how and why it comes to be.

## CHAPTER I: Ramanujan and Rediscovery

#### *Realist Narratives*

Consider the following passage, written by an Indian mathematician:

[Euler and Jacobi] had the advantage of a complete university education behind them and unlike Ramanujan they went through the trodden path of academics to acquire their reputation. But Ramanujan did not have this good fortune of a formal university training. He became a mathematician before anybody could think of training him. It is well that he was not trained early because it is debatable whether he could have been so prolific if he had been trained to watch every mathematical step of his. One may say that he was 'uneducated' if I may be permitted to use that word, compared to an Euler or Jacobi or for that matter any mathematician in the world. Ramanujan was a self-taught genius. He could dispense with all the technical elaborations of the 18<sup>th</sup> and 19<sup>th</sup> century mathematics and still have much to say, which continues to occupy the attention of several mathematicians in the world.<sup>65</sup>

This passage portrays Ramanujan heroically facing off against generations of European mathematicians, and winning. Conveying the same sentiment is a passage by Ramanujan's chief Indian biographer, Ranganathan: "The thought created in the West had not even been disseminated in the country [India]. Everything had to be done and discovered by him *de novo*."<sup>66</sup> Hardy, though, bemoans Ramanujan's struggle as tragic, not heroic. "He had been carrying an impossible handicap, a poor and solitary Hindu pitting his brains against the

<sup>&</sup>lt;sup>65</sup> V. Krishnamurthy, "Srinivasa Ramanujan: A Biographical Sketch and a Glimpse of his Work," *Srinivasa Ramanujan Centenary* (Special Issue of the Journal of the Indian Institute of Science, Bangalore, 1987): 11.

<sup>&</sup>lt;sup>66</sup> S. R. Ranganathan, *Ramanujan: The Man and the Mathematician* (Bombay: Asia Publishing House, 1967), 21.

accumulated wisdom of Europe. He had no real teaching at all; there was no one in India from whom he had anything to learn."<sup>67</sup> For Hardy, rediscovery is irrelevant, because it adds nothing new to mathematical knowledge. That so much of Ramanujan's work had been anticipated meant Ramanujan had tragically wasted his most productive years.<sup>68</sup>

The Indian hagiographical accounts might raise our suspicions about the extent to which Ramanujan was, entirely by himself, equal to more than a century's worth of the greatest mathematical minds of Europe. Hardy gives us one possible response: we reject valuing Ramanujan as a way to prove India equal to Europe and value Ramanujan as an individual instead.

But a deeper underlying problem us gives cause to be skeptical about both the Indian accounts and Hardy's perspective. There are very specific philosophical implications if Ramanujan managed to rediscover mathematics in isolation and without the aid of teachers, collaborators, or references: such a narrative is strong support for mathematical realism, or Platonism.<sup>69</sup> The philosophical roots of realism were expressed by Plato in the *Timaeus*: "That which is apprehensible by thought with a rational account is the thing that is always unchangeably real; whereas that which is the object of belief together with unreasoning sensation is the thing that becomes and passes away, but never has real being."<sup>70</sup> 'Unchangeably real' as

 <sup>&</sup>lt;sup>67</sup> G. H. Hardy, *Ramanujan: Twelve Lectures on Subjects Suggested by his Life and Work* (Providence, RI: AMS Chelsea Publishing, 1999), 10. A development of two lectures delivered at the Harvard Tercentenary Conference of Arts and Sciences in the fall of 1936. First Edition, Cambridge, 1940. Cited hereafter as Hardy, *Twelve Lectures*.
 <sup>68</sup> Hardy, *Twelve Lectures* 6-7. Sixteen years earlier, Hardy wrote that had Ramanujan been properly trained he

might "have been less of a Ramanujan, and more a European professor, and the loss might have been greater than the gain..." but Hardy here dismisses his previous statement as "quite ridiculous sentimentalism."

<sup>&</sup>lt;sup>69</sup> There are different brands of realism; what I refer to here is specifically *realism in ontology* (see below), which is often called "Platonism." Cf. Stewart Shapiro, "Philosophy of Mathematics and its Logic: Introduction," *The Oxford Handbook of Philosophy of Mathematics and Logic*, ed. Stewart Shapiro, <u>3-27</u>. (Oxford University Press, 2005), 6.
<sup>70</sup> Plato, *Timaeus*, trans. Francis M. Conford (New York: Macmillian Publishing Company, 1959), 16.

opposed to that which 'becomes and passes away' suggests the existence of eternal truth, accessible through a 'rational account,' i.e., rational thought. This implies that the rational reflection of two unconnected individuals in different times and places will necessarily yield the same conclusions given the same premises. In the *Metaphysics* Aristotle critiques an ontologically developed version of this theory, a theory he ascribes to Plato and his followers and calls the theory of Forms.<sup>71</sup> Forms are abstractions that exist as metaphysical entities, not as constructs of the human mind, and are independent of and prior to physical objects or human conception.<sup>72</sup>

A quasi-mystical connection to the mathematical realm is exactly what underpins the conceptions of mathematics of Ramanujan's Indian commentators, and possible Ramanujan himself (see next paragraph). This is Platonism with a capital "P." Although obviously the specifics of this quasi-mystical connection are not the same as in Plato, the basic sentiments are the same, so we can cite Plato as a familiar historical precedent.

However, while I introduce realism through a discussion of Platonism, I do not adopt use of 'Platonism.' This is because I wish to discuss the ontological realism that I argue underpins Hardy, Nandy and Kanigel as well, and none of these writers hold quasi-mystical views. Nandy explicitly rejects mysticism, Kanigel gives mysticism the benefit of the doubt but does not affirm it, and Hardy argues against a mystical interpretation of Ramanujan. But insofar as none of these writers actively reflected on their philosophical assumptions, they did not have developed and consistent philosophical views. Not only did they not have any epistemology in place of an ad-hoc mystical explanation to explain how we are able to access mathematical objects, but also even their basic ontology is underdeveloped. They have no details about the nature of the existence of mathematical objects, only the assumption that mathematical objects are 'real' in that they are external to human beings. Because Plato also did not leave us a developed ontology (the ontology we have is that which Aristotle records as the position of Plato), he is an excellent way to capture the basic philosophical sentiments of Hardy, Nandy, and Kanigel, even if Platonism is ultimately not an accurate label.

Note that these are descriptive categories, not actors' categories, as all these writers have few or no analytical philosophical reflections themselves. Hardy identifies himself as a realist, while admitting both that the study of the philosophy of mathematics is a subject that demands expertise of which he has none, and that "I know too well how probable it is that just the most sympathetic philosophies will prove untenable." G. H. Hardy, "Mathematical Proof," *Collected Papers of G. H. Hardy* Vol. XII, 581-585.

<sup>&</sup>lt;sup>71</sup> Aristotle *Met.* XIII. 4-5. J. L. Ackrill, *A New Aristotle Reader* [Selections], (Princeton, N.J.: Princeton University Press, 1987), 355-357.

<sup>&</sup>lt;sup>72</sup> I do not wish to imply that this intellectual heritage comes directly from Plato, unbroken and unchanged down to the modern day. My purpose here is twofold. First, few modern definitions or discussions of Platonism explain how it derives from Plato and why it should carry his name, and I find this an interesting etymological and historical point. Second, the form of Platonism I wish to discuss bears a great resemblance to Plato's original doctrines. Shapiro (6) writes, "The connection with Plato [in the term 'Platonism'] might suggest the existence of a quasimystical connection between humans and the abstract and detached mathematical realm. However, such a connection is denied by most contemporary philosophers. As a philosophy of mathematics, 'platonism' is often written with a lowercase 'p,' probably to mark some distance from the master on matters of epistemology. Without this quasi-mystical connection to the mathematical realm, the ontological realist is left with a deep epistemic problem. If mathematical objects are in fact abstract, and thus causally isolated from the mathematician, then how is it possible for this mathematician to gain knowledge of them?"

A theme we will repeatedly encounter is how realism underpins positions that seem to contradict each other. For example, both the dedicated atheist Hardy and the observant Brahmin Ramanujan were ardent mathematical realists (Hardy explicitly stated this, but for Ramanujan we have only secondhand accounts). This is perhaps why, after six years of working with Ramanujan, Hardy was surprised when Aiyar and Rao wrote that Ramanujan had "definite religious views."<sup>73</sup> Hardy and Ramanujan worked together for six years with a common realism underlying their conception of mathematics, and Hardy was subsequently not able to conceive how a shared realism could underpin the contradictory positions of atheism and religiosity.<sup>74</sup>

That is, assuming Hardy was wrong and Ramanujan was religious; but reconstructing Ramanujan's views from surviving sources is a perilous process. The people who give testimonials about Ramanujan's mystical views are the same ones who then try to present Ramanujan as a mystic. It is possible that these sources present Ramanujan as holding mystical views in order to legitimate their own personally held beliefs. It is also possible that these Indian authors, coming as they did from cultural backgrounds similar or identical to that of Ramanujan, had the same views as Ramanujan. Without any further sources we can say little more about Ramanujan, but we will continue to examine the sources themselves.<sup>75</sup>

<sup>&</sup>lt;sup>73</sup> Aiyar and Rao, "Srinivasa Ramanujan (1887—1920)," *Collected Papers of Srinivasa Ramanujan*, ed. G. H. Hardy, P. V. Seshu Aiyar, and B. M. Wilson (Cambridge: The University Press, 1927), xviii. Hereafter, I will cite this biography as Aiyar and Rao, *Collected Papers*.

<sup>&</sup>lt;sup>74</sup> Kanigel has a different interpretation. He argues that Hardy's emotional distance intimidated Ramanujan and prevented him from discussing anything personal with Hardy. Hardy was oblivious to this; and in reflecting on Ramanujan's life, Hardy thought that Ramanujan's silence on personal and religious matters meant that he did not care about them. Kanigel 288.

<sup>&</sup>lt;sup>75</sup> We do have one account that is not from a suspect individual, that of C. P. Mahalanobis, although his account is quoted in Ranganathan. Nandy (127-128) very helpfully provides background about Mahalanobis, writing, "The reminiscences of Mahalanobis are again pertinent here, not only because he knew Ramanujan at Cambridge, but also because as an agnostic, Marxist, mathematical statistician, he can be expected to ignore the magical interpretations which adulatory biographers like Ranganathan try so hard to foist on us." We can still suspect that

We can consider the streak of mysticism running through the accounts of Ramanunjan's Indian commentators as ontologically equivalent to realism. While the authors probably did not self-identify as realists, when they claim that Ramanujan's mathematics originated from a supernatural source, they identity mathematics with the ontological properties of the supernatural source. Mathematics, like the supernatural, is universal, eternal, and prior to the physical realm. The only major distinction with realism is that these accounts suggest that spontaneous, intuitive access to mathematics is superior to mathematics accessed from a methodological, rational process.<sup>76</sup> But epistemological issue of *access* is independent of the ontological issue of *existence*. The *ontology* of the Indian accounts is equivalent to realism.

Aiyar and Rao express a mystical view when they write, "Ramanujan used to say that the goddess Namakkal inspired him with formulæ in his dreams. It is a remarkable fact that frequently, upon rising from bed, he would note down results and rapidly verify them..."<sup>77</sup> This is quoted speculatively. But when we examine an earlier obituary written by Rao, we see that at least Rao took the supernatural assertions seriously: "And he [Ramanujan] is no more. Is he? So

Ranganthan has edited, censored, or selectively quoted Mahalanobis, but we will trust that Ranganathan has not fabricated it outright.

According to Mahalanobis' account, Ramanujan connected mathematical objects to fundamental metaphysical concepts of his Vedic/Brahmin philosophizing. Cited in Ranganathan 82, Mahalanobis recalls, "He sometimes spoke of 'zero' as the symbol of the Absolute (*Nirguna-Brahmam*) of the extreme monistic school of Hindu philosophy... He looked on the number 'infinity' as the totality of all possibilities, which was capable of becoming manifest in reality and which was inexhaustible... Each act of creation, as far as I could understand, could be symbolized as a particular product of infinity and zero..."

Since Mahalanobis' account accords with the other testimonials, we can take those more seriously as a group.

However, individual accounts do not become any more reliable. Even if an individual source concurs with Mahalanobis' account, it is not enough to certify its reliability, as the 'reliability' we care about comes from context and not content. Hence we still do not have enough reliable evidence by which we could comprehensively reconstruct Ramanujan's conception of mathematics. But from Mahalanobis' account, we can at least make a minimum assertion that Ramanujan's mysticism, like that of other authors discussed below, is equivalent to ontological realism. In Ramanujan's case, if mathematical objects are equivalent to universal and external metaphysical objects, then mathematical objects are universal and external as well.

<sup>&</sup>lt;sup>76</sup> Plato's original realism doesn't fully distinguish between the intuitive and rational, as his concept of 'rational' does not imply rejection of the metaphysical. Plato's 'rationality' is transcendental and heavily mystical.

<sup>&</sup>lt;sup>77</sup> Aiyar and Rao, *Collected Papers*, xii.

far as he is concerned, if there be any truth in his own beliefs—such beliefs, as he himself acknowledged, led him intuitively to his discoveries—he is still existing and has only quitted a world of shams for a world of realities."<sup>78</sup> Rao suggests that the success of Ramanujan's mathematical discoveries is evidence for the truth of Ramanujan's purported mystical and religious beliefs, beliefs consisting of distinguishing 'a world of realities' from 'a world of shams.' The implicit connection between mathematics and the 'world of realities' suggest that this 'world of realities' is the source of mathematical discoveries. Again, while not directly realism, the assumptions about the ontological properties of mathematical objects are close enough for us to treat this as realism.

Ranganathan similarly has a realist ontology embedded in his mysticism. He writes that only intuition, and not intellect, can lead to 'Total Apprehension [of reality].'<sup>79</sup> Elsewhere he writes, "Surely, all this [Ramanujan's work from 1907 to 1911] could not have been seized by the intellect alone. Intuition should have played a large part in this period of super-activity. Ramanujan was indeed a *Drashta* (= a Seer) in Mathematics."<sup>80</sup> Intuition is sufficient but not necessary for mathematics; and, intuition is necessary but not sufficient for Total Apprehension. The confluence does not directly equate mathematics with Total Apprehension, but does closely associate them.

Hardy readily identifies himself as a realist. In one speech, he professes, "It seems to me that no philosophy can possibly be sympathetic to a mathematician which does not admit, in one manner or another, the immutable and unconditional validity of mathematical truth.

<sup>&</sup>lt;sup>78</sup> Dewan Bahadur R. Ramachandra Rao, "In Memoriam: S. Ramanujan," *Journal of the Indian Mathematical Society* XII, (3) (Madras: Srinivasa Varadachari & Co., June 1920): 88-89.

<sup>&</sup>lt;sup>79</sup> Ranganathan 95-97.

<sup>&</sup>lt;sup>80</sup> Ranganathan 22.

Mathematical theorems are true or false; their truth or falsity is absolute and independent of our knowledge of them. In *some* sense, mathematical truth is part of objective reality... Lagrange discovered [that 'any number is the sum of 4 squares'] in 1774; when he discovered it he discovered *something*; and to that something Lagrange, and the year 1774, are equally indifferent.<sup>"81</sup>

Obscuring the shared ontology are explicit rhetorical clashes between these perspectives. Hardy rejects the parts of Aiyar and Rao's biography that suggest Ramanujan was at all religious or a mystic, convinced that his understanding of Ramanujan as a secular and rational agnostic is the correct one (despite Hardy having no full conversation with Ramanujan about religion in the six years they knew each other).<sup>82</sup> Ranganathan argues against Hardy, claiming that Hardy did not understand the nature of religious experience.<sup>83</sup> Nandy is not shy about dismissing both the "magical interpretations which adulatory biographers like Ranganathan try so hard to foist on us"<sup>84</sup> and the "aggressively positivist Hardy."<sup>85</sup> Nandy is particularly brutal on Hardy. Hardy had based his appraisal of Ramanujan's agnosticism on one statement Ramanujan made that all religions seemed to him more or less equally true. To this, Nandy points out that Ramanujan's statement is actually a part of the prescribed daily prayer of a Brahmin, and hence anybody "with

<sup>&</sup>lt;sup>81</sup> G. H. Hardy, "Mathematical Proof," *Collected Papers of G. H. Hardy* Vol. XII, 584. Hardy continues, "We have done no more than to make explicit a few of the instinctive prejudices of the 'mathematician in the street'. Yet with our first demand [unconditional validity of mathematical truth] we have antagonized at least two-thirds of the philosophers in the world; and with the second [that the 'something' of Lagrange has (to put Hardy's words into more technical language) independent ontological existence] we have reduced our first indiscretion to entire insignificance, since we have committed ourselves, in one form or another, to the objective reality of our propositions, a doctrine rejected, I believe, not only be all philosophers, but also by all three of the current schools of mathematical logic [logicism, formalism, intuitionism]."

<sup>&</sup>lt;sup>82</sup> Hardy, *Twelve Lectures*, 5.

<sup>&</sup>lt;sup>83</sup> Ranganathan 100.

<sup>&</sup>lt;sup>84</sup> Nandy 128. Also, the "over-determined mystical explanations of Ranganathan." Nandy 144, n. 20.

<sup>&</sup>lt;sup>85</sup> Nandy 119. Nandy's use of 'positivist' does not seem appropriate here, he perhaps means 'realist.' The confusion might be the emphasis of both on basing knowledge on logic and rationality.

even a cursory acquaintance with Hindu religious systems will immediately see the absurdity of [Hardy's] conclusion."<sup>86</sup>

Nandy's strong rhetoric hides the extent to which he shares the assumptions of those he critiques. Nandy accepts the narrative of an isolated, self-taught genius,<sup>87</sup> and takes for granted that "Ramanujan depended very little on the existing structure of mathematics, and his work can hardly be called a logical development of the formal mathematical knowledge of his time. His exposure to modern mathematics was next to nothing...<sup>388</sup> While Nandy disputes the notion that mathematics is "the only 'true' science with no determinants outside its own world," what his skepticism of realism amounts to is arguing that determinants affect our *access* to this mathematical world. Determinants have no affect on that world itself: "To say that pure mathematics, unlike most other human pursuits, is culture-free and unencumbered by human

<sup>&</sup>lt;sup>86</sup> Nandy 119. Note that Brahminism is in many ways extremely relativistic, and has been centuries before modernism (which is probably what has made Vedantic philosophy so attractive a source for the 'new-age' movement or any relativists seeking legitimizing historical precedents). But the complete story is that this philosophy is coupled with and used to support an extreme institutional absolutism, where any criticism of orthodoxy or the caste hierarchy is relativistically neutralized with scholastic-like, self-satisfied, one-sentence rhetorical dismissals. For an influential example, see Swami Dayananda Saraswati, *Light of Truht [sic] or An English Translation of Satyarth Prakash*, trans Dr. Chiranjiva Bharadwaja (New Delhi: Sarvadeshik Arya Pratinidhi Sabha, 1975). First published in 1915, this is an early and canonical statement of Hindu nationalism, a movement that in many ways is the modernization of Brahminical absolutism. An excellent study and critique of the way relativism plays out in the justifying Hindu nationalist absolutism in Indian politics is Meera Nanda, *Prophets Facing Backward: Postmodern Critiques of Science and Hindu Nationalism in India* (New Brunswick, N.J.: Rutgers University Press, 2003). For a general study of the history of the Hindu right, see Christophe Jaffrelot, *The Hindu Nationalist Movement in India* (New York: Columbia University Press, 1996).

<sup>&</sup>lt;sup>87</sup> Nandy 94. Also, Nandy x: " Srinivasa Ramanujan (1887-1920) [is] probably the greatest untrained genius mathematics has produced in recent centuries. When he was accidentally discovered at the age of twenty-five, he was working as a clerk in Madras on a salary of twenty-five rupees a month. But on his own, helped by a hopelessly backdated and second-rate text-book for undergraduates, he had rediscovered some of the major discoveries made in mathematics during the previous hundred years and was in many fields far ahead of his contemporaries." Nandy (3) also writes that what attracted him to consider writing about Ramanujan in the first place was Hardy's statement about Ramanujan pitting his brains against all of Europe – hence Nandy's approach was shaped from the beginning by the idea of an opposition of cultures.

<sup>&</sup>lt;sup>88</sup> Nandy 95

emotions is to say that only some kinds of persons can become creative mathematicians."<sup>89</sup> Nandy does not see time, place, or community as relevant factors. Mathematics is independent of these; the only thing it is not independent of is an individual's psychology. Like the other accounts, Nandy is an ontological realist about mathematics.

Kanigel is not as dramatic a case, as he does not directly contest other sources. He does argue that Hardy was wrong about Ramanujan, but argues that it was because of Hardy's lack of ability to communicate with Ramanujan on a personal level and not because of any philosophical concerns about the role of rationality. He presents the Indian accounts of the mystic and religious Ramanujan only to argue that mysticism and his Brahmin religion were an integral part of his life. He implicitly disagrees with the Indian accounts when he says that Ramanujan's religious beliefs do not explain his mathematical talent,<sup>90</sup> but avoids providing his own conjecture. He only makes a soft and uncontroversial claim about Ramanujan's religion and mysticism contributing to his creativity. Still, Kanigel's account has an underlying realism. Kanigel accepts as unproblematic the many accounts of Ramanujan's rediscovery (which I discuss extensively below); he quotes E. T. Bell's highly realist pedagogical suggestion about leaving students to fend for themselves; and most importantly, he portrays Ramanujan as learning mathematics on his own and doing mathematics on his own, with other people serving only an incidental and non-essential role in Ramanujan's acquisition of mathematical knowledge. Social, cultural and personal factors are crucial only insofar as they retard or prevent access to mathematical knowledge; they play no crucial role in the creation of that knowledge.

<sup>&</sup>lt;sup>89</sup> Nandy, 94-95.

<sup>&</sup>lt;sup>90</sup> Kanigel 284. However, the Indian accounts do not imply that anybody with religious beliefs would have mathematical talent. They argue that a mathematical talent as unique as that of Ramanujan's requires a supernatural explanation, and use Ramanujan's personal beliefs to support a specific supernatural explanation.

Why does realism underlie the accounts of all of these writers, having as they do such different and often contradictory priorities and concerns? The commonality between these is that all these accounts use the figure of Ramanujan to accomplish some goal or argue some point. And the way these arguments have formed, they rely on Ramanujan being *special*. The more special he is, the better he serves the author's purpose.<sup>91</sup> When realism connects Ramanujan to a universal and eternal world, it dissociates him from his context and surroundings. Ramanujan's knowledge becomes a function of his personal relationship and connection to the realm of mathematical forms, trivializing his social ties and place in his society.

Here we revisit constructivism, discussed in the introduction. Constructivism is not the only alternative to realism, but it contrasts with realism in clear and distinct ways. Constructivism is an approach that describes knowledge based on the circumstances of its production. While realism separates knowledge from context, constructivism makes knowledge a product of context. And when applied to individuals, realism atomizes and elevates the individual, while constructivism blends individuals together and prioritizes communities. Because my own approach is constructivist, I will explore how the goals of the various biographers, formulated as they are, would be ill served by constructivism. This will allow us to see the extent and importance of realism in these accounts.

Part of Nandy and Kanigel's realism might be incidental. The majority of the nonphilosophical or amateur philosophical discourses about mathematics are loosly realist, and so it has become the default for new non-philosophical or amateur philosophical discourses. It is not obvious why realism matters (indeed, this is something I am arguing for), and without any

<sup>&</sup>lt;sup>91</sup> Unlike my argument, where the less special Ramanujan is, the better he serves my purpose.

reason to go against the default, an author may wind up espousing realism without really knowing it. Probably only the mystical accounts of Ramanujan strictly *need* realism to accomplish their goal (because taking the supernatural seriously requires some sort of realist ontology). For the other accounts, they could attain their goal with a project and approach not based on realism. But for on the projects Kanigel and Nandy have created, realism is essential.

Nandy's expressed goal is to show that "Ramanujan's life allows one a glimpse into the alternative world of 'what could have been' without the interventions of the dominant culture of science and the dominant mode of scientific socialization." This goal becomes meaningless if Ramanujan was a member of the dominant culture and dominant mode. In a constructivist account, in order to produce mathematics resembling that of the 'dominant' British culture, Ramanujan would need to be a part of, or connected to, that culture. But the realist assumptions about mathematical knowledge being universally and eternally accessible allow for the possibility of a non-British mathematics whose results are nonetheless isomorphic with British mathematics. This is exactly what Nandy seeks: 'alternative science.' He seeks to use Ramanujan to establish that this is indeed possible. As Nandy stresses, it is not an "Indian model of science" that he supports; he wants us to "speculate about a future science which might allow one to integrate the speculative, normative, and aesthetic factors with the logical, rational, and empirical ones." 92 He wants a *universal* science that is not the result of a historical process, and not the unique cultural product of the west. Realism allows him to challenge the western hegemony without being antiscience, and without relativistically claiming the equality of some Indian tradition to modern

<sup>&</sup>lt;sup>92</sup> Nandy 94-5, 14

science; with realism, he can argue that what he seeks already exists in a universal essence of science.

For Kanigel, realism is an immediately attractive perspective. Realism, with all of its transcendent implications, makes possible vast, romantic and dramatic portrayals. I would argue that realism has been so resilient, since Plato, in part because it has tremendous aesthetic appeal. A realm of eternal and universal truths offers the prospect of *certainty*, and the transcendent perfection of this realm inspires a sense of wonder. Realism can excite us while comforting us. Any view related to relativism, in contrast, will involve inherent instability, limits to knowledge and understanding, and underlying disorder and arbitrariness. The aesthetic<sup>93</sup> value of relativism will only be greater than that of realism when relativism gives a more satisfying account of the world. Until the arrival of evidence that undermines the ability of realism to account for the world, there is no reason to consider relativism.

Ranganathan seems to have two related concerns. He is an Indian nationalist and a devout Hindu, and he wants to validate both of these. In Ramanujan he finds a hero for India's present, a validation of India's past, and a model for India's future, all expressed in religious terms. For India's past, Ranganathan presents Ramanujan as the reincarnation of centuries of dormant Indian mathematical ability. India's mathematical prowess of centuries ago never disappeared, it only slumbered, and Ramanujan now vindicates its former greatness. For India's future, Ranganathan holds up Ramanujan as the ideal man, exemplifying humility and virtue, industriousness and devotion. For India's present, he holds up Ramanujan as an example of a

<sup>&</sup>lt;sup>93</sup> I mean *aesthetic* in the sense the world being justified only as an aesthetic phenomenon (*Birth of Tragedy* §5). When I say one view is aesthetically preferable to another, I do not mean that it is preferable when considered in the abstract. The most important determinant of the aesthetic value of a perspective is how well it can account for our experience of reality.

man inspired by divine grace, showing that such grace is present and active in India. As mentioned above, overtly mystical and religious accounts that maintain the existence of supernatural forces require some form of realist ontology; here there is no possible constructivist alternative.

Hardy is a bit harder to understand, because he has no easily discernable 'project' like the other authors. Hardy is a professional mathematician; he is neither writing a story like Kanigel, nor trying to reconcile cultures like Nandy, nor promoting a religious and nationalist agenda like Ranganathan. Neither could he have been a realist for lack of knowing any alternatives; he was well aware of the discussions in philosophy of mathematics going on in Cambridge at the time, and actively chose not to study them and to keep making his philosophical reflections as an amateur.<sup>94</sup> All possible motivations would reduce to a desire for self-validation or external, social validation, which is well into the territory of psychology. I think psychological history is for the time being is doomed to be unsatisfactorily speculative, because of the lack of a systematic model of the mind that could make such analyses not be ad-hoc. From Hardy's writings, we see some evidence that his commitment to realism was aesthetic, but his choice to deliberately remain an amateur suggests some deeper personal motives (especially as he expressed having no tolerance for amateurs in his own field).<sup>95</sup>

Out of the accounts we consider, realism helps the authors accomplish their goal in a way that a constructivist account could not. But a constructivist account has a further inherent difficulty; if there exists a Platonic realm of mathematics, rationally accessible across time and

<sup>&</sup>lt;sup>94</sup> G. H. Hardy, "Mathematical Proof," Collected Papers of G. H. Hardy Vol. XII, 581-585.

<sup>95</sup> Ibid.

space, then it would be no surprise that Ramanujan managed to tap into it and thus rederive generations' worth of European mathematics without any access or exposure to it. But under a constructivist view, where mathematical knowledge would be the result of and unique to a specific time, place and community, Ramanujan appears to be a baffling anomaly or at best an unbelievably fantastic coincidence. While social constructivism deals easily with the independent discovery that indeed is common in science and mathematics, constructivist explanations rely on the existence of a common community. Within a scientific community, practitioners share nearly identical technical backgrounds and pedagogy, and are usually in communication with other members of the community. If two scientists share steps 1 through 99 in common (to borrow Hardy's favored metaphorical measure of a hundred), are working towards similar or identical goals, and then independently arrive at a similar or identical step 100, we should not be surprised. However, we would be incredulous at the suggestion that the two scientists could start at step 1 and independently converge on the same steps 2 through 100 without there being an underlying deterministic process, the steps of which exist prior to either scientist.

My own motivation for giving a non-realist treatment of Ramanujan is based on Ramanujan having become the perfect example of realism.<sup>96</sup> The whole realist belief of the possibility or even necessity of independent discovery in isolation is seldom tested in the extreme, as there are so few cases where a person emerges from genuine isolation to make significant contributions – perhaps in modern memory no example other than Ramanujan, but certainly no greater example. In this sense, for an aesthetic argument about the appeal of realism versus a

<sup>&</sup>lt;sup>96</sup> For example, Edward Shils builds a case for realism by relying on the example of Ramanujan. Edward Shils, "Reflections on Tradition, Centre and Periphery and the Universal Validity of Science: the Significance of the Life of S. Ramanujan, *Minerva* 29 (4) (Dec 1991): 391-419.

relativist, constructivist perspective to give an account of science and mathematics, Ramanujan is a crucial test case. In an analytic sense, showing that the case of Ramanujan can be equally or better explained by a constructivist account does not disprove realism, but denying realism its poster boy does remove a significant aesthetic argument for the desirability of a realist perspective.

In the remainder of this chapter, I seek to destabilize the realist narrative of Ramanujan as an isolated auto-didact by contesting the assumptions and treatment of evidence of the realist accounts. I will do this through two interrelated techniques. First, I analyze some cases that are quoted as "independent discovery," and show not so much their lack of independence, but that these discoveries are not so unambiguously independent. Second, I demonstrate the state of mathematical knowledge around Ramanujan; when relevant, I show that circumstances existed in which Ramanujan could potentially have had access to information that current biographers assume was impossible. Where possible, I connect the two techniques by showing how a specific 'independent discovery' could have come from a specific channel of exposure. Some of what I do is to make novel connections, but much of this project only requires emphasizing marginalized aspects of existing biographies (especially detailing the mathematical community that surrounded Ramanujan).

#### Ramanujan's First Rediscovery

For our first mathematical examination, consider the following anecdote, quoted by every biographical source as an example of Ramanujan's rediscovery. This is from Aiyar and Rao, who were the first to give this story: While in the fourth form [approximately tenth grade], he took to the study of Trigonometry. He is said to have borrowed a copy of the second part of Loney's *Trigonometry* from a student of the B.A. class, who was his neighbor. This student was struck with wonder to learn that this young lad of the fourth form had not only finished reading the book but could do every problem in it without any aid whatever; and not infrequently this B.A. student used to go to Ramanujan for the solution to difficult problems. While in the fifth form, he obtained unaided Euler's Theorems for the sine and cosine and, when he found out later that the theorems had been already proved, he kept the paper containing the results secreted in the roofing of his house.<sup>97</sup>

Can this story be genuine? Presumably, the theorem of Euler meant is the famous identity  $e^{i\theta}$  = sin  $\theta$  + *i* cos  $\theta$ . But that is given, with a derivation, on page 76 in part 2 of Loney.<sup>98</sup> Ramanujan's biographers sometimes manage this discrepancy, although none explicitly acknowledge it. Kanigel divorces the two parts of Aiyar and Rao's account, mentioning Ramanujan's convergence with Euler (p. 50) independently of Ramanujan's receipt of Loney's *Trigonometry* (p. 27). In citing Ramanujan's receipt of Loney, Kanigel writes in an endnote that "Most likely, according to Richard Askey [a mathematician at the University of Wisconsin], this was only part 1 of the two-volume text."<sup>99</sup> Hardy provides a very creative resolution, preserving the integrity of both parts of the story by reversing the chronological order. "[Aiyar and Rao] say, for example, that soon after he had begun the study of trigonometry, he discovered for himself 'Euler's theorems for the sine and cosine' (by which I understand the relations between the circular and

<sup>&</sup>lt;sup>97</sup> P.V. Seshu Aiyar and R. Ramachandra Rao. "Srinivasa Ramanujan (1887-1920)," in *Collected Papers of Srinivasa Ramanujan*. G.H. Hardy, P.V. Seshu Aiyar and B.M. Wilson, eds. pp. *xi-xii*.

<sup>&</sup>lt;sup>98</sup> S.L. Loney. *Plane Trigonometry*, 2<sup>nd</sup> ed. Vol. 2. Cambridge: At the University Press, 1895. pp. 76.

<sup>&</sup>lt;sup>99</sup> Kanigel 378.

exponential functions), and was very disappointed when he found later, apparently from the second volume of Loney's *Trigonometry*, that they were already known."<sup>100</sup>

We immediately note two things here: first, the likelihood that this claimed rediscovery took place is actually far from certain, and second, discrepancies are completely marginalized by biographical sources. This shows how realism reinforces itself; a realist treats suggestions of rediscovery uncritically and unproblematically, which then produces a narrative that reinforces the perception that a realist epistemology best explains mathematics.

This is not a very important case because we have no direct evidence for this story, unlike later cases where Ramanujan's notebooks provide a detailed record of Ramanujan's mathematical output. However, it is an excellent historiographical case study because it provides us with two lessons. The primary lesson of this story lies in how the same reasons that make this story almost certainly apocryphal serve as an argument against the realist assumptions about the inevitability of rediscovery. The secondary lesson is the evidence related to the story, far from showing that Ramanujan was in conditions of isolation where he could make 'independent' rediscovery, shows that Ramanujan was actively learning and receiving resources from people around him.

In terms of which books Ramanujan could or could not have seen, my own inclination is to agree with Richard Askey insofar as Ramanujan could not have seen part 2. Here I follow the same lines as Askey in comparing the contents of the different parts of Loney with Ramanujan's work. Part 2 of Loney is concerned in large part with complex numbers, something that does not appear anywhere in Ramanujan's notebooks (except edited in later in a different ink, presumably after he was in England learning from Hardy). Since the textbook Ramanujan would

<sup>&</sup>lt;sup>100</sup> Hardy, *Twelve Lectures*, 2.

subsequently rely on, Carr's *Synopsis* (discussed below), also lacked any treatment whatsoever of analytic functions, <sup>101</sup> and even hardly any entries on algebra with complex numbers, it is reasonable to suppose that Ramanujan never saw any significant treatment of complex numbers.<sup>102</sup> This would not have just been a matter of taste, as complex numbers were crucial to some things Ramanujan worked on. For example, Ramanujan was led astray when he mistakenly treated the Riemann zeta-function as though all roots were real.<sup>103</sup> But if we conclude from Ramanujan's ignorance of complex quantities that Ramanujan did not see part 2 of Loney, it does not mean that he did indeed independently rediscover Euler's Theorem; rather, Ramanujan would not have even been familiar with the terms that would allow him to make sense of Euler's equation! As surprising as this possibility sounds, it is no more surprising than Ramanujan's lack of knowledge of complex functions. Hence, this story about rediscovery is almost certainly apocryphal.

The primary lesson here is that Ramanujan's ignorance of not just analytic functions but even of the significance of complex numbers is not compatible with realist assumptions about

<sup>&</sup>lt;sup>101</sup> When we look up things like complex quantities, imaginary numbers, and analytic functions in Carr's index, we see that he almost never discusses them in his book – the extensive index entries he gives refer almost entirely to outside sources, including French and German texts, which of course would have been no trouble for a Cambridge student to procure. (Analytic functions are basically well-behaved complex functions of a complex variable  $f: \mathbb{C} \rightarrow \mathbb{C}$ . I have referred to Lars V. Ahlfors, *Complex Analysis: An Introduction to the Theory of Analytic Functions of One Complex Variable*, 3<sup>rd</sup> ed. New York: McGraw-Hill, Inc. 1979, §1.2, p. 24).

he discusses the binomial theorem for complex quantities, i.e. the expansion of  $(I + z)^n$ , "when z is complex ( = x +  $y\sqrt{(-I)}$ )." Loney's discussion of the binomial expansion for complex quantities could have at least suggested the existence of complex roots to Ramanujan.

<sup>&</sup>lt;sup>103</sup> Hardy, *Twelve Lectures*, 9-10.

rediscovery. That such a simple and fundamental topic, relevant and essential in the areas in which he was working, could have eluded Ramanujan suggests that Ramanujan was constrained by boundaries that preclude the possibility of independent rediscovery. In other words, this undermines the realist conception about the 'naturalness' of a particular line of reasoning, a consequence of the Platonic connections that exist between various pieces of information. If we were to think about complex numbers as a 'natural consequence' of any of the many areas of mathematics that connect to it, this case shows that such a notion only exists in retrospect. Or, to put it differently, to quote from Littlewood a perfect example of realist reasoning: "If the architect of the Taj Mahal had designed a different tomb for Arjumand Banu, the most beautiful building in the world to-day would not have come into existence. But if Euclid had not discovered that the number of prime numbers is infinite, nevertheless that same theorem would have been discovered long ago by somebody else."<sup>104</sup> If we had not had complex numbers, it seems as though it could not have been Ramanujan who would have discovered them.

The secondary lesson is about access to mathematical knowledge beyond just a given textbook. Note who reportedly gave the textbook to Ramanujan: his neighbor, a college student studying mathematics. While Aiyar and Rao report that it was Ramanujan's neighbor, Kanigel (perhaps mistrusting their overall narrative) prefers to synthesize other accounts and suggests that it was college-going boarders, which Ramanujan's family would house for some extra money, who gave Ramanujan the textbook. Either way, Ramanujan would have had *direct and constant domestic access* to individuals who were receiving structured, institutionalized mathematical education. I find it hard to conceive, then, that he did not receive something like the equivalent

<sup>&</sup>lt;sup>104</sup> Neville, in *Essays and Surveys* 109.

of private tutorship from them; based on his reported enthusiasm for mathematics, unfettered domestic access would have enabled him to constantly pester them to teach him more. Ramanujan was enormously self-motivated if he worked through a book like part I Loney, but he was not self-taught; most likely, he received tacit knowledge from college students about how to work through Loney and perhaps direct guidance as well.

Ramanujan's ignorance of complex functions also reflects on the limitations of this community. The community was not a perfect or complete source of mathematical knowledge, but it was a source on which Ramanujan probably relied. For example, in their obituary, Iyer and Spring remark that "Ramanujan learned from an older boy how to solve cubic equations."<sup>105</sup> Furthermore, the proximity of college-age students was not a coincidence: despite Ramanujan being from a rather small township, Kumbakonam's Government College was less than a twenty minutes' walk from Ramanujan's high school.<sup>106</sup> Ramanujan would later briefly attend this college, whose mathematical resources were far from nonexistent.

## Carr and Ramanujan's Rediscovery of Bernoulli

It was from the nearby collegiate community, or possibly from an 'elderly friend,'<sup>107</sup> that Ramanujan in 1903 received the much-discussed *Synposis of Elementary Results in Pure and Applied Mathematics* of George Shoobridge Carr. In a touch of irony, the index begins with a page devoted to a quote from Professor J. D. Everett, "There is an immense amount of knowledge lying scattered at the present day, and almost useless from the difficulty of finding it

<sup>&</sup>lt;sup>105</sup> Letters and Commentary 206. In an obituary written by Narayana Iyer and Sir Francis Spring.

<sup>&</sup>lt;sup>106</sup> Kanigel 45.

<sup>&</sup>lt;sup>107</sup> Ranganathan 19-20.

when wanted."<sup>108</sup> For, as much as Carr contained, many topics were left for the student to look up from external sources cited in a Joint Index of the *Synposis* and 'Papers on Pure Mathematics' – including analytic functions.

The very fact that Carr came into Ramanujan's possession contains some evidence about the kind of information that was available to Ramanujan. Carr was an unusual book; Hardy says that the book was an obscure work, the coaching notes of a Tripos tutor.<sup>109</sup> Presumably, Carr would not have any thought to circulate the work in India, because it was a book tied to a thing, the Tripos, which was geographically associated with Cambridge. In Masters of Theory, Andrew Warwick comments the uniqueness of Cambridge pedagogy: as one former Trinity Undergraduate who went on to teach mathematics at the University College London realized, he was not able to fit the materials developed around Cambridge pedagogy into the curriculum of this other schools.<sup>110</sup> That the book came into Ramanujan's possession testifies to the randomness, but existence, of channels by which British mathematical knowledge filtered down to Ramanujan. While it was a lowly book in that it was a supplementary study guide created for 'teaching to the test' of the Tripos, and not designed as a general or independent pedagogical tool, the Synposis was also a lofty book, because the rote and tedium it worked through corresponded exactly to what some of the top mathematicians in the world were learning at Cambridge.111

<sup>&</sup>lt;sup>108</sup> Carr 842.

<sup>&</sup>lt;sup>109</sup> Hardy, *Twelve Lectures*, 2-3.

<sup>&</sup>lt;sup>110</sup> Andrew Warwick, *Masters of Theory: Cambridge and the Rise of Mathematical Physics* (Chicago: University of Chicago Press, 2003), 151.

<sup>&</sup>lt;sup>111</sup> Here the difference between mathematics and mathematical physics in the Tripos becomes apparent. As Warwick (24-26, 228-229) describes, the gap between the most difficult Tripos questions and actual research was sometimes very small, sometimes even with unpublished research results appearing on the exam. This made the exam quite

While it is unlikely that Ramanujan could have understood novel mathematical topics from the book alone, when supplemented with the sporadic tacit mathematical knowledge he received from individuals around him, it was enough for Ramanujan to become a competent mathematician. Despite Carr's omissions of large chunks of mathematics, terse style devoid of proofs, and antiquation, Carr was still a topical overview of Cambridge mathematical education. Then, as Ramanujan came into contact with individuals of increasing mathematical prestige, and was able to absorb more and more tacit mathematical knowledge, by the time he contacted Hardy he was producing work that was, for all its strangeness, recognizable in form and content to the then-top Cambridge mathematician, Hardy.

Our second mathematical examination focuses on the Bernoulli numbers, and relates predominantly to historiographical issues of presentation and emphasis. Kanigel presents this account: "Ramanujan had stumbled on Bernoulli numbers for the first time about eight years before [i.e., 1903], though probably without having ever heard of them as such. The second volume of Carr's *Synopsis* contained references to them in various guises, but Ramanujan may not have seen it until 1904, when he was at Government College – a year after he apparently began working with them. In any case, he'd worked with them ever since…"<sup>112</sup> Ramanujan, as

effective at preparing students for research in mathematical physics. Hardy, however, puts blame squarely on the Tripos for the retardation of British pure mathematics, saying that the style it encourages is completely disconnected from research. G. H. Hardy, "The Case Against the Mathematical Tripos (Presidential Address to the Mathematical Association, 1926)," *Collected Papers of G. H. Hardy* Vol. VII (Oxford: Claredon Press, 1979), 350-351. While Hardy's rhetoric often outpaced his insight and understanding, Hardy's success by 1909 in his efforts to reform the Tripos shows the traction of his critique. Warwick (434) writes that after these 1909 reforms, the tradition of mathematical physics was marginalized, and the pure mathematics research of people like Hardy and Littlewood became increasingly popular.

<sup>&</sup>lt;sup>112</sup> Kanigel 90.

Berndt points out, had a great love and affinity for Bernoulli numbers,<sup>113</sup> and his first published paper (in 1911) was about properties of Bernoulli numbers.

One 'guise,' as Kanigel puts it, in which Bernoulli numbers appear is a direct definition in entry 1539 in Carr. This is followed by several entries of derivations involving the numbers (and also including the Euler-Maclaurin summation formula some pages earlier).<sup>114</sup> Not only that, but in Berndt's edit of Chapter III of Ramanujan's second notebook, in the place where Ramanujan first introduces Bernoulli numbers, Berndt makes the comment that Ramanujan defines Bernoulli numbers in a way different from the current standard – and Ramanujan's definition is none other than the one given in Carr.<sup>115</sup> I doubt Berndt meant here that Ramanujan came up with this definition (as in the corresponding page in the facsimile copies, Ramanujan does not define Bernoulli numbers and only uses  $B_2$ ,  $B_4$ ,  $B_6$ , "&c" without comment);<sup>116</sup> Berndt likely is quoting Ramanujan's choice of definition from elsewhere, or perhaps even from Carr (though it is not cited here).

There is still the possibility that Ramanujan discovered Bernoulli numbers for himself earlier, but subsequently learned the standard notation and employed that when copying his results into his second notebook. The difficulty is that Ramanujan's work involving Bernoulli numbers does not appear in the corresponding place in Ramanujan's first notebooks. They may still be hidden away somewhere in the rest of the volume. Or, even if they are, as we do not have precise datings for the notebook, Ramanujan still might have started working with Bernoulli numbers only after he had seen Carr's definition. In any case, Kanigel's conjecture, made

<sup>&</sup>lt;sup>113</sup> Berndt, Notebooks 1, 7.

<sup>&</sup>lt;sup>114</sup> Carr 276.

<sup>&</sup>lt;sup>115</sup> Berndt, Notebooks I, 51.

<sup>&</sup>lt;sup>116</sup> Tata Institute Facsimiles Volume II, 27.

without citation, seems his own speculation; both that Ramanujan had 'stumbled' upon them, and that Ramanujan had not seen the second volume of Carr until he was at Government College. Kanigel's distinction between the two volumes of Carr is also odd; while Carr is a twopart work, there does not seem to be any issue of the two parts being separate like Loney. Furthermore, differential calculus only begins from volume 2, so if Ramanujan were working on this topic he would have already been going through the second volume.

While this might well be innocent speculation on Kanigel's part, it is another instance of realist assumptions reinforcing and propagating themselves. Kanigel, seeing Ramanujan as the great isolated rediscoverer, perhaps assumed that he not only *could* have but *would* have rediscovered Bernoulli numbers for himself. Then, when we read Kanigel's account of Ramanujan rediscovering Bernoulli numbers before seeing them in Carr, we get the impression that this was yet another of many instances of rediscovery in isolation.

### Independent Discovery in Isolation, Independent Discovery in a Community

For our third and final examination of Ramanujan's mathematical resources, we look ahead to the time when Ramanujan had access to the wider mathematical community. After failing out of two colleges consecutively (for neglecting all subjects aside from mathematics), he managed to impress a wealthy member of the newly-formed Indian Mathematical Society, who then privately funded him for a short time. It was during this period that his first published paper appeared in the *Journal of the Indian Mathematical Society*. Ramanujan subsequently took a job as a minor clerk, but after he began his famous communication with G. H. Hardy, officials in Madras arranged for him to have a research scholarship at the University of Madras. From his two brief stints in college, he had begun to interact more with a community of professional mathematicians in India (we will explore this community fully in Chapter 2). Here we examine what Ramanujan's widening access to mathematical resources, both human and print, means for potential rediscovery.

First, there were library resources: Hardy notes that "there had been periods of his life when he had access to the library in Madras, but it was not a very good one; it contained very few French or German books; and in any case Ramanujan did not know a word of either language."<sup>117</sup> Here we cite K. R. Rajagopalan's biography of Ramanujan. Rajagopalan reports that Ramanujan claimed he would scan French and German texts, following the arguments from whatever mathematics he could understand.<sup>118</sup> Rajagopalan's account is the most questionable I cite; but the claim is reasonable, and offers a possibility of how Ramanujan could have picked up sporadic knowledge.

Such sporadic knowledge can contain far more than it might seem. The knowledge of a community of practitioners is necessarily encoded in the various pieces of information they provide. Webs of relationships connect a work like a textbook to everything that has come before, even if the textbook does not explicitly contain all previous results. For example, Carr could not have been unaware of complex numbers; he made a pedagogical decision to exclude them. But then the gamma function he gives is not a gamma function that only applies to the positive reals, but the general statement of the function, for which it applies to the complex plane as well. He prefaces the formula saying that the argument is "a real and positive

<sup>&</sup>lt;sup>117</sup> Hardy, *Twelve Lectures*, 10.

<sup>&</sup>lt;sup>118</sup> Rajagopalan 13.

quantity."<sup>119</sup> If one were to think to apply this to the complex plane as well and discover that it worked, one would not have independently discovered an extension of the gamma formula to the complex plane, but only unpackaged information implicitly contained within the specific formulation of the function. In such a manner, knowledge not explicitly represented can be encoded.<sup>120</sup>

While stated generally as such, this proposition is reasonable, but I would need to be able to demonstrate specific encodings in order to be able to contest Berndt's statement that, "Quite remarkably, Ramanujan independently discovered a great number of the primary classical theorems in the theory of hypergeometric series. In particular, he rediscovered well-known theorems of Gauss, Kummer, Dougall, Dixon, Saalschütz, and Thomae, as well as special cases of Whipple's transformation."<sup>121</sup> But sometimes such encoding is not needed to explain supposed independent discovery of classical theorems. First we must remember that while Ramanujan used his notebooks as evidence of his productivity when visiting professors and seeking patrons, Ramanujan never intended to publish them. He likely did not pay attention to making sure that each result he was recording was original. For example, Ramanujan presents Gauss's continued fraction in the same way as his original work, as part of an unbroken list of results, but Berndt notes that this is contained explicitly in Carr.<sup>122</sup> Hence we have at least this one instance of Ramanujan not tracking attribution, and perhaps not even being aware of having

<sup>&</sup>lt;sup>119</sup> Carr §2284, pg 359.

<sup>&</sup>lt;sup>120</sup> In arguing for the persistence of previously produced knowledge, even when a community might not be aware of it, an important precedent is Augustine Brannigan, "The reification of Mendel," *Social Studies of Science* 9 (4) (Nov. 1979): 423-54. Brannigan shows how Mendel's work, supposedly rediscovered, had a persistence presence. While this discussion focuses around an entire community, similar amnesia might be at work in recalling what Ramanujan may have known about.

<sup>&</sup>lt;sup>121</sup> Berndt Notebooks II, 7

<sup>&</sup>lt;sup>122</sup> Berndt Notebooks II, 134. Carr p. 97

previously seen some theorem that he derived. While this is only one case, I again make the appeal to shift the presumption away from independent rediscovery of classical results.

One perfect example of the strength of the existing level of presumption comes from Berndt:

Formula (42.1) [<sup>123</sup>] is due to C. B. Halderman [<sup>124</sup>]. It is quite remarkable that Ramanujan used the same notation as Haldeman and recorded the terms in precisely the same order as Haldeman! One might conclude that Ramanujan had seen Haldeman's paper or a secondary source quoting it. However, this seems highly unlikely, for Ramanujan had access to very few journals in India, and, moreover, Haldeman's paper was published in a very obscure journal. It is also quite possible that Ramanujan made his discovery before Haldeman did. Thus, in conclusion, the identical notation must be an amazing coincidence.

Likewise (42.2) [<sup>125</sup>] is also due to Haldeman [p. 289]. A. Martin [<sup>126</sup>] also found a proof of (42.2), but, inexplicably, did not acknowledge Haldeman's priority. However, at the beginning of his paper, Martin remarks that "Mr. Cyrus B. Haldeman of Ross, Butler, Co., I., contributed a valuable paper 'On biquadrates' which was published in the present volume of the Magazine, pp. 285-296." (The title of Haldeman's paper is slightly misquoted.) Ramanujan does not use Haldeman's notation but does employ Martin's notation in (42.2). Again, this must be an astonishing coincidence.<sup>127</sup>

 $<sup>^{123}(8</sup>s^{2} + 40st - 24t^{2})^{4} + (6s^{2} - 44st - 18t^{2})^{4} + (14s^{2} - 4st - 42t^{2})^{4} + (9s^{2} + 27t^{2}) + (4s^{2} + 12t^{2})^{4} = (15s^{2} + 45t^{2})^{4}$ 

<sup>&</sup>lt;sup>124</sup> "On biquadrate numbers," The Math. Mag. 2 (1904), 285-296. pp. 289, 290

 $<sup>{}^{125} \</sup>left(4m^2 - {\rm I2}n^2\right)^4 + \left(3m^2 + 9n^2\right)^4 + \left(2m^2 - {\rm I2}mn - 6n^2\right)^4 + \left(4m^2 + {\rm I2}n^2\right)^4 + \left(2m^2 + {\rm I2}mn - 6n^2\right)^4 = \left(5m^2 + {\rm I5}n^2\right)^4$ 

<sup>&</sup>lt;sup>126</sup> "About biquadrate numbers whose sum is a biquadrate–II," *The Math. Mag.* 2 (1904), 325-352. pp. 325, 326, 331 <sup>127</sup> Berndt, *Notebooks* IV, 95

First, I note that part of the purpose in forming the Indian Mathematical Society was to collectively buy and circulate foreign journals among its members,<sup>128</sup> and second, I cite a personal recollection of Ranganathan:

In November 1913, I was the only student in the post-graduate honours class in Mathematics of the Madras Christian College. [Edward B.] Ross was my only Professor... One day he entered the class-room with his eyes glittering and his lips throbbing. He asked me, "Does Ramanujan know Polish?" I replied that it was not at all likely. The Professor said, "Even if he did, it will make no difference." I was puzzled by this laconic remark of the Professor. Then he pulled out from his pocket a university envelope stuffed with a bunch of sheets. He threw the sheets open before me and said, "This is the quarterly report of Ramanujan as a research student of the University. Look at this beautiful theorem. In the issue of a Polish periodical brought by today's mail, something of this kind appears. Surely, Ramanujan could not have divined what that Polish mathematician was thinking. What is more, Ramanujan's theorem is much deeper. Ramanujan has certainly anticipated the Polish mathematician. He is extraordinary. Is he not?"<sup>129</sup>

Ranganathan gives this story as another one of Ramanujan's victories over Europe; but what I draw attention to here is that, according to Ranganathan's account, a foreign journal in an

<sup>&</sup>lt;sup>128</sup> From the first issue of the *Journal*, all periodicals received and books published were listed. The first issue lists the purchase of books such as Hardy's Course of Pure Mathematics, Love's Theoretical Mechanics, and Askwith's Analytical Geometry of the Conic Sections. The periodicals include the American Journal of Mathematics, the Mathematical Gazette, the Messenger of Mathematics, Proceedings of the Edinburgh Mathematical Society, Proceedings of the Royal Society of London, Transactions of the American Mathematical Society, and Transactions of the Cambridge Philosophical Society. The *Journal* established rules by which these materials could circulate among members. M. T. Naraniengar and S. Narayana Aiyar, eds. *Journal of the Indian Mathematical Club* I (1-3) (Madras: S. Murthy & Co., The "Kapalee" Press).

<sup>&</sup>lt;sup>129</sup> Ranganathan 12.

obscure language was available in India to an individual who knew Ramanujan and presumably knew Polish. Again, while records of every mathematical scrap Ramanujan might have come upon are obviously not preserved, we see the many opportunities for surprising bits and pieces of mathematics do filter down to Ramanujan.

This story of Ramanujan supposedly anticipating a Polish mathematician also leads to my next point. My argument for embedded knowledge only applies to knowledge produced in the far past, the work of Euler and Gauss and Jacobi. It does not apply to contemporary convergence. However, Ramanujan's convergence with mathematicians of his own time does not weaken my argument, but reduces to a previously studied phenomenon. Kuhn noted in the Structure of Scientific Revolutions that nearly simultaneously discovery is common in science, but in all cases this happens within the context of a continuous community connecting the multiple independent discoverers. This is the constructivist argument; if Ramanujan was indeed a member of a contemporaneous mathematical community, and not a figure in isolation, the convergence he had with other mathematicians is not staggeringly unusual. Only rediscovery of classical results in isolation would be inexplicable under a constructivist account. A contemporary community is formed from tacit bonds that require interpersonal contact and communication; an equivalent connection can never exist with the static material of the past, because the kind of understanding that facilitates simultaneous independent discovery cannot be rationally formulated.

### Conclusion

The reader may now judge whether two-thirds of Ramanujan's best work in India being rediscovery, as Hardy estimated, is still more probable than Ramanujan having somehow accessed a given piece of mathematical knowledge. This is considering not only Ramanujan's sporadic access to unexpected literature, but also to mathematical *pedagogy* in the form of interpersonal communication. While this never conclusively proves that Ramanujan saw a given piece of literature, I hope the reader will agree with me that this treatment destabilizes the image of Ramanujan in isolation, and makes us question the necessity of the realist perspective in explaining Ramanujan's mathematical work.

By comparing Ramanujan's access of mathematical knowledge to that of Cambridge mathematicians, we also get a reasonable explanation of how Ramanujan knew the mathematics he did, but was ignorant of professional standards of presentation. While there are many differences between exams and research, the presentation style of research mathematical physics closely resembled the format of exam questions. Warwick presents the example of J. H. Poynting, whose 1883 paper presenting the Poynting vector presented his work in a logical progression that was identical to the exam style Poytning had been trained in at Cambridge, but very different from the thought process that actually led Poynting to the result. Ramanujan's haphazard training resembled the pedagogical setup of Tripos coaches, but after failing out of college, all of Ramanujan's creative energies were not focused towards goal of preparing for an exam of a very specific format.

Lectures, tutorials, textbooks, graded examples and written examinations are such the norm in scientific training today that we barely give them a second thought. But, Warwick writes, these devices did not exist before the late eighteenth and nineteenth centuries. He notes that the seemingly trivial material culture of pen and paper actually came as a strange novelty when it began to replace oral methods of demonstrating knowledge. Associating scientific training and research respectively with undergraduate and graduate study, located at a University, also dates to no more than two centuries ago.<sup>130</sup>

Finally, readers should also note that Ramanujan's emergence coincided directly with the arrival of institutions of British higher education in India. This, as well as further descriptions of other figures related to Ramanujan's life, follow in Chapter 2.

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<sup>&</sup>lt;sup>130</sup> Warwick 26, x.

# CHAPTER 2: Context and Continuity

#### Expanding Networks

The conventional portrayal of Ramanujan was that he was an unprecedented prodigy who languished in isolation until he sent a letter with some of his mathematical work to G. H. Hardy. Hardy saw the genius of the work, and proceeded to lift Ramanujan out of obscurity and bring him to Cambridge. The two had a brilliant collaboration that ensured Ramanujan's proper place among the greatest mathematicians for all time.

What is not satisfying about this portrayal? There are three things that strike us as fantastic. First, that Ramanujan could emerge unexpectedly, without precedent in his community. Second, that Ramanujan's genius went unrecognized for so long. Third, that Ramanujan's career skyrocketed just as soon as G. H. Hardy entered the picture. The implausibility of these occurrences provides on one level a sense of wonder, but on a deeper level, a sense of great dissatisfaction.

To take an example of how an existing biography tackles one of these mysteries: Ranganathan proposes that the fact Ramanujan was such an extreme outlier<sup>131</sup> was not a matter of chance.

It is not possible to explain the phenomenon of Ramanujan except on the hypothesis of the ever-increasing *Purvajanma-vasana* — the Psycheo-genetic force — gaining in momentum all through the march of a soul from embodiment to embodiment... enough statistical data ha[s] not yet accumulated about the frequency

<sup>&</sup>lt;sup>131</sup> Two younger brothers survived Ramanujan, but they remain in almost complete obscurity because, interestingly enough, they seem to have been rather lazy and unremarkable, and certainly had no special aptitude for math. Nandy writes about Ramanujan's close relationship with his mother, and this gives us a possible explanation; Ramanujan's mother lavished all her time and attention on Ramanujan's development, such that his younger brothers never got proper nurture.

of appearance of such phenomenal men of genius, and of the social and hereditary factors bearing on them, to make possible the formulation of any empirical law regarding the subject.<sup>132</sup>

In other words, Ramanujan was the reincarnation of the concentrated karmic potency of centuries of dormant Indian mathematical ability.

Ranganathan is at least right that pure chance is not a very convincing explanation of how Ramanujan might be such an extreme outlier. But we can take another approach entirely; instead of trying to explain how Ramanujan was so unique, we can question the extent to which he was truly unique. For if he was not such an outlier, then there is no need for Ranganathan's mystical conjecturing.

We can think of the fantastic aspects of Ramanujan's life as 'discontinuities.' While 'discontinuity' is an artificial category, it is one we often rely on in personal understanding, and hence we will use it to talk about our perceptions. We think of a 'discontinuity' as when a single event causes massive changes and reorganizations. It is not immediately obvious, but discontinuities can only happen when something external disrupts a previously closed system – as in the case of perhaps the introduction of a fully developed technology in the form of a conquering army.<sup>133</sup> Because, *within a closed system, discontinuity is impossible*. What we see in retrospect as a major change will always have had intermediary steps leading up to it.

Internal discontinuities are impossible, yet because of their ability to enact vast and fantastic changes, we want to believe in them. We want to believe that Ramanujan could have emerged from nowhere to challenge the British hegemony in mathematics. We want to believe

<sup>&</sup>lt;sup>132</sup> Ranganathan 17.

<sup>&</sup>lt;sup>133</sup> Lynn White Jr., "The Medieval Roots of Modern Science and Technology," *Medieval Religion and Technology: Collected Essays*, Chapter 5 (Berkeley: University of California Press, 1986).

that an incredible outlier can emerge at any time, in any place. We want to believe in that sudden, vast validation and elevation of reputation can happen.

In this chapter, I argue against three perceived discontinuities in Ramanujan's story: discontinuity in historical events, discontinuity in institutional connections, and discontinuity in individual lives. These correspond to the mystery of Ramanujan's long obscurity, the fantastic nature of Ramanujan's sudden discovery by Hardy, and to the unimaginable distance between Ramanujan and the circumstances from which he emerged. We will see the continuity in historical events by observing how Ramanujan had been gaining greater and greater recognition in the time before he contacted Hardy, and how there were causal connections between one recognition and the next. The continuity of institutional connections is established by exploring the channels by which Ramanujan's reputation increased, and seeing how the channel through which Ramanujan contacted Hardy closely resembled previous, halfway successful attempts at gaining recognition. Channels of communication at this time were based on previously made personal acquaintances linking institutionally unconnected people, especially within the context of the British Empire. We will see the continuity in individual lives by considering other Indians who were studying mathematics, other Indians who were going to study in England, and even another Tamilian who went to study pure mathematics at Cambridge like Ramanujan.

This is partly just a shift in emphasis. Even though Kanigel mentions all of the relevant actors in Ramanujan's life, it is a question of how he presents them. He writes,

During most of the ten years since he'd encountered Carr's *Synopsis*, Ramanujan had inhabited an intellectual wilderness... [he was] a mathematical genius of perhaps once-in-a-century standard cut off from the mathematics of his time. He roused wonder and admiration among those, like Narayana Iyer and

Seshu Iyer, who could glimpse into his theorems. Yet no one had been able to truly appreciate his work. He had been alone. He had no peers.<sup>134</sup>

Elsewhere, Kanigel describes the close mathematical collaboration between Ramanujan and his superior officer at his clerkship, Narayana Iyer. Kanigel cites stories of how the two would work together on mathematics until the early hours of the morning. But Kanigel frames such descriptions with statements like that above that trivialize and dismiss the importance and relevance of Narayana Iyer. We will attempt not just to mention relevant actors and argue that they had an impact on Ramanujan, but to remove Ramanujan from the center of the discourse to which everything relates, and present patterns into which he fits.

# Continuity of Networks

By 1912, the most important person in Ramanujan's mathematical life was S. Narayana Iyer (1874-1937), his superior at the clerkship job he secured. Narayana Iyer was the son and grandson of Brahmin astrologers, and he remained religious – never in his life adopting western dress – but took a different career path, earning an M.A. in mathematics at St. Joseph's College in Trichinopoly, and staying there some time as a lecturer.<sup>135</sup> But he did not go into a career of mathematics, first joining the Public Works Department in Madras in 1900, and then joining the Madras Port Trust as Office Manager upon the request of Sir Francis Spring. At the Port Trust, he was promoted to Chief Accountant, making him the highest-ranking Indian there. He was a founding member of the Indian Mathematical Society, in which he served as assistant

<sup>&</sup>lt;sup>134</sup> Kanigel 210.

<sup>&</sup>lt;sup>135</sup> Letters and Commentary 73-74, and Bruce C. Berndt, "A Short Biography of S. Narayana Iyer," in *Ramanujan: Essays and Surveys*, pp. 97-98.

secretary and, later, as treasurer. He also was a strong advocate of Ramanujan to Sir Francis Spring<sup>136</sup> – important because Spring was still unconvinced of Ramanujan's merit.

Narayana Iyer and Ramanujan developed a close working and personal relationship. They would work together on mathematics at Iyer's house until late into the night.<sup>137</sup> While we do not have more details about the content of their collaboration, we do have two papers that Iver published in the Journal of the Indian Mathematical Society in 1913, the entire first have half of the second devoted to communicating results of Ramanujan's. All the information in the first paper, "The Distribution of Primes," was also in Ramanujan's second letter to Hardy. In about the middle of the second paper, Iver gives what Ramanujan called his "Master Theorem" (see below), and even remarks on the constraints of the rigorousness of this theorem, writing, "in the form as given by Mr. Ramanujan... we have to proceed with some caution as the transition from the penultimate step to the last step will not be strictly true in the case of some periodic functions" and then giving an outside reference.<sup>138</sup> From this we can see first that Iyer was fully involved with the intricacies of Ramanujan's work, and hence their mathematical relationship must have been very close, and second that Iyer understood the necessity of conditions that Ramanujan skipped over; hence, as much as Ramanujan's skill might have surpassed his, Iyer had a tremendous amount to teach Ramanujan.

While all communication with Hill was managed by Griffith and Spring, Ramanujan's letters to Hardy are seen as having been from Ramanujan. While this is true, Ramanujan later wrote to Hardy, "all letters written to you, except this one and the remainder, did not contain

<sup>&</sup>lt;sup>136</sup> Letters and Commentary 75

<sup>&</sup>lt;sup>137</sup> Ranganathan.

<sup>&</sup>lt;sup>138</sup> Iyer's two papers, given in Essays and Surveys: 100, 103-4. Berndt's commentaries on Iyer's papers, 100, 104.

my language. Those were written by the superior officer mentioned before [Iyer] though the mathematical results and handwriting were my own."<sup>139</sup> Interpreting this has become a contested point. Ranganathan takes the arrogance of the letters as evidence that Ramanujan did not write them. After all, "the shyness and humility of Ramanujan were un-exampled. The correct term to describe this quality of Ramanujan is the Sanskrit word *Hri* in the sense in which Valmiki used it in evaluating Rama."<sup>140</sup> Kanigel disagrees with Ranganathan; Kanigel points out that Ramanujan was always very conscious to acknowledge help from others, observed later in the papers he published with Hardy, and that the letter in which he acknowledged help was not very different in tone from the previous letters, and concludes that Ramanujan was present in the letters.<sup>141</sup> Kanigel, in the only place in which he portrays Ramanujan as anything but wondrous, here is probably explicitly trying to humanize him by finding some token faults, for which he selects a lack of humility (and even this is portrayed in the softest light possible).<sup>142</sup> I think that far more important than the input of Narayana Iyer into the arrogance of the letter is that the supposedly

<sup>&</sup>lt;sup>139</sup> Letters and Commentary 90. Iyer is not named, but Ramanujan earlier referred to a superior officer who was a "very orthodox Brahmin." However, Ramanujan cites this orthodox superior as having immediately discounted the possibility of him going to England; one source suggests that Ramanujan presents this in order to avoid responsibility for his initial refusal.

<sup>&</sup>lt;sup>140</sup> Ranganathan 32. Here Ranganathan's biography literally becomes hagiography, as Ranganathan equates Ramanujan in all his aspects with Rama, the traditional model of the ideal human being, and then holds up Rama/Ramanujan as the example to be followed. The legendary King Ram or Rama who is the focus of the great Indian epic the Ramayana was God incarnate (by birth he was an incarnation of a fourth of Vishnu, but later in the Ramayana, Rama is told that he is the ultimate reality). His reverence in Indian society was displayed when the RSS and radical Hindu nationalist groups were able to exploit devotion to Lord Ram to incite a mob to tear down the Badshahi Mosque, which supposedly stood on the birthplace of Lord Ram, and had been built over a temple destroyed by Mughals. The incident, of course, had nothing to do with archeology (which was absent) and everything to do with putting Muslims in their place.

<sup>&</sup>lt;sup>141</sup> Kanigel 385

<sup>&</sup>lt;sup>142</sup> When Kanigel (175-178) presents Ramanujan's arrogance, he does so by juxtaposing first an extremely flattering and somewhat contrary quote of Neville, arguing that Neville's absence of mentioning humility meant that this one virtue Ramanujan did not possess: "E. H. Neville would later describe Ramanujan as 'perfect in manners, simple in manner, resigned in trouble and unspoilt [sic] by reknown, grateful to a fault and devoted beyond measure to his friends.' Nowhere, though, did he call him humble, or suggest that Ramanujan shrank from a sanguine assessment of his own gifts."

'self-evident' mathematics done and presented independently by Ramanujan was actually closely supervised by a professionally trained mathematician. Also more significant for us than Ramanujan's virtues is noting what this co-authorship suggests about the circumstances in which the letter was conceived. Iyer, despite his orthodoxy, later strongly encouraged Ramanujan to take the opportunity to go to Cambridge;<sup>143</sup> and considering this, the idea or initiative to write the initial letter to Hardy may well have been on Iyer's recommendation.

## Continuity of Events

Networks would prove to be not only institutionally important for Ramanujan, but epistemologically as well. We can view in two ways the long purgatory in which Ramanujan's work was passed from mathematician to mathematician with reserved endorsement. We could say that Ramanujan's work remained the same throughout, and that only external factors like individual incompetence or bias prevented Ramanujan's work from being recognized sooner. Or, we could propose that Ramanujan and his work were not static, that Ramanujan's strategies and presentation as well as the content of his work evolved during the time he attempted to gain recognition – Ramanujan's travel through his networks changed and refined the content of his mathematics. The former perspective, held by the existing biographical accounts, puts tremendous focus on Hardy as the key variable, which is reasonable but still quite remarkable. If we treat as relevant the intermediate steps where Ramanujan won provisional or partial recognition, we see a much more reasonable story of an increasing curve, not a spike, charting the relevant points of the increase of Ramanujan's perceived legitimacy.

<sup>&</sup>lt;sup>143</sup> Essays and Surveys, 98.

During a period of joblessness, Ramanujan made contacts with several members of the Indian Mathematical Society, who helped him get a clerkship. He worked at the Accountant General's Office, a division of the Madras Port Trust, a major colonial operation headed by Sir Francis Spring (1849-1933). Spring was a graduate of Trinity College, Dublin, and major colonial figure (previously instrumental in developing the South Indian Railways System).<sup>144</sup> It was not long before Ramanujan explicitly came to Spring's attention; Ramanujan started in March 1912, and in November, C. L. T. Griffith, graduate of University College, London, and Professor of Civil Engineering at Madras College of Engineering, wrote to Spring saying, "He may be a very poor accountant, but I hope you will see that he is kept happily employed until something can be done to make use of his extraordinary gifts."145 The connection between Ramanujan and Griffith is unclear, but Ranganathan says that it was upon the request of Ramanujan's earlier benefactor (and later biographer) Ramachandra Rao that Griffith wrote to Spring.<sup>146</sup> Griffith writes in this letter, "Our Math. Professor here says that very few people could follow or criticize this work."147 Griffith adds that he is writing to someone working in the same field as Ramanujan at home, but until he hears back, "I don't feel sure that it is worth while spending much time or money on him," The person to whom Griffith wrote was M. J. M. Hill (1856-1929), Griffith's old Professor of Mathematics at University College, London. Griffith sent Ramanujan's first published paper on Bernoulli numbers, and probably some amount of unpublished work as well.<sup>148</sup>

<sup>&</sup>lt;sup>144</sup> Letters and Commentary, 13.

<sup>&</sup>lt;sup>145</sup> Letters and Commentary, 12.

<sup>&</sup>lt;sup>146</sup> Ranganathan.

<sup>&</sup>lt;sup>147</sup> Letters and Commentary, 13.

<sup>&</sup>lt;sup>148</sup> Letters and Commentary, 17.

Spring then seems to have sought the opinion of Sir Alfred Bourne, F.R.S. (1859-1940), another graduate of University College, London, and a Professor of Biology at Presidency College. Borne had not heard of Ramanujan, and treats the various judgments of possible genius skeptically, saying, "if his genius is so elusive or mysterious that good mathematicians possessed besides of much common sense cannot recognise and appreciate it even if it carries them beyond their scope, I should doubt its existence."<sup>149</sup> Borne's first recommendation would be Richard Littlehailes (1878-1950), a former Professor of Mathematics at Presidency College, but Littlehailes was away on the west coast. Instead he recommends that Spring send Ramanujan to Middlemast, or Graham.

Graham was J. F. Graham (1879-1916), trained in Mathematics and Mathematical Physics at Trinity College, Dublin, currently appointed to the central colonial administration of the region at Fort St. George. We have a letter from Graham to Griffith, where Graham again recommends Middlemast, and again gives an ambiguous judgment that Ramanujan seems to have skill and be doing interesting original work, but it is also possible he was a calculating prodigy of the idiot savant type. Griffith passes this on to Spring on November 28<sup>th</sup>, noting that Middlemast had already been consulted and likewise was unsure. Griffith recommends waiting for Prof. Hill's response.

Hill received Griffiths' November 12<sup>th</sup> letter on December 1<sup>st</sup>, and mailed a quick reply two days later to catch the next mail shipment, sending a more complete reply on December 7<sup>th</sup>. Unfortunately, Hill's first reply was mostly negative, although not categorically dismissive; he says Ramanujan had "fallen into the pitfalls of the very difficult subject of Divergent Series," and

<sup>&</sup>lt;sup>149</sup> Letters and Commentary, 13-14.

that Ramanujan should read Bromwich's *Theory of Infinite Series*. Beyond this Hill says little, adding that if were to Ramanujan send polished, complete work to the Secretary London Mathematical Society his results would be published. The reply is not entirely generic, as Hill specifically references Ramanujan's use of  $c_1$ ,  $c_2$ ,  $c_3$ , ... without saying what they are, and a reference to a §12 that does not exist, as mistakes Ramanujan needs to correct.

Although a setback in the immediate frame, this development was an important expansion of Ramanujans' network. With Hill we again have an explicit Cambridge connection, this time an even more illustrious one, as he had been 4<sup>th</sup> Wrangler in 1879. Also important to note that spearheading Ramanujan's communication with mathematicians in England was not Ramanujan the solitary clerk, but by the most qualified and eminent British mathematicians in the area acting on Ramanujan's behalf. The tone of the letters often seems as though these figures seek only to satisfy their own curiosity; certainly, they are hardly *advocating* for Ramanujan or his legitimacy. But the very fact that they were entertaining Ramanujan, and going out of their way to do so (Griffith had not had any contact with Hill for two decades, so this was not a casual correspondence), was effectively advocacy – and at that, more influential advocacy than anything Ramanujan had received prior.

Furthermore, even though Griffith and Spring were investigating Ramanujan's ability not at his request but on their own initiative, Ramanujan was not entirely an external object; when Hill implicitly addresses Ramanujan in his replies (in language like, "if Mr Ramanujan will write out his ideas carefully and clearly... and send them to the Secretary of the London Mathematical Society..." and "he should not use symbols with he does not explain...")<sup>150</sup>, and when Griffith and Spring share Hill's replies with Ramanujan, Ramanujan becomes a member of the conversation. Hill's criticisms of Ramanujan are not just negative judgments in reply to Spring and Griffith's query, but constructive criticism for the benefit of Ramanujan.

Berndt and Rankin write that Ramanujan had definitely written to Hardy before seeing Hill's second letter; then Hill's reply would have been eclipsed by that of Hardy and not played any role in future developments.<sup>151</sup> But the letter tells us some interesting things, even though in terms of substantial support for Ramanujan it contains little more than the first letter. Hill gives a more detailed criticism of Ramanujan's marginal or absent proofs, and repeatedly stresses Bromwich, writing, "I do not think you can do better for him than get him a copy of the book I recommended, Bromwich's Theory of Infinite Series, published by Macmillan and Co., who have branches in Calcutta and Bombay... If this book could be obtained for him, and he would work at it <u>from the beginning</u> [emphasis original], it would be much better for him than for those who are interesting in him to spend money on printing his papers at the present time... And I hope that after Mr. Ramanujan has read Br. Bromwich's book he will be able to write something which will stand the test of modern criticism."<sup>152</sup> Hill more charitably adds that divergent series have tripped many 'illustrious mathematicians of earlier days,' and that when

<sup>&</sup>lt;sup>150</sup> Letters and Commentary, 16.

<sup>&</sup>lt;sup>151</sup> Letters and Commentary , 16. In particular, none realized Ramanujan and Hill had a shared interest in hypergeometric series, which was not among the material Griffith sent to Hill, and would have potentially elicited much more interest from Hill. In fact, Ramanujan's notebooks contain a more general and stronger result of material Hill published in 1907 and 1908 (again, note that the convergence is contemporary), but only eighty years later Berndt discovered this in examining Ramanujan's notebooks, and nothing came of this connection. <sup>152</sup> Letters and Commentary, 18-19.

Hill was a student in Cambridge, they were not well understood, with the modern theory having only recently been established.

There are a few things of note here. First is that Hill constantly speaks of procuring Bromwich for Ramanujan, and not urging Ramanujan to get Bromwich and assuming that Ramanujan himself should take on this responsibility, assuming or suggesting that Ramanujan was worth at least this measure of continuing patronage. Second, we note the effect of the delay in post communication between Britain and India, about a month between a given individual sending a letter and receiving a reply. Third, even early on here we have Griffith making an observation with exactly the sentiment of many future evaluations of Ramanujan's work, writing in a note he attached when forwarding Hill's letter to Spring, "The fact that Prof. Hill is prepared to consider Ramanujan's Elliptic Functions, even though the proofs may not be of that logical completeness that is demanded now-a-days shows that it is possible that the 'intuitive results' may be of interest."<sup>153</sup>

From this we see that prior to contacting Hardy, Ramanujan had been increasingly coming to the attention of prominent mathematicians in India, and now beginning in England. Ramanujan's contacting Hardy was a natural progression in strategy, and Hardy's enthusiastic response was a progression of results.

## Continuity of Individual Lives: The Context of Foreign Travel

We can also look at the founding of mathematics departments at various Indian universities. The founding of an independent department would represent not the arrival of a

<sup>&</sup>lt;sup>153</sup> Letters and Commentary, 19.

subject, but a critical point of interest and faculty that would justify putting funds towards creating an entire department. The University of Calcutta founded a mathematics department in 1916, the University of Allahabad in 1922, and the University of Madras in 1927. The latter we might be surprised about, that the prestige associated with Ramanujan did not translate into a mathematics department being founded at Madras earlier, but the university as a whole was oriented towards arts and languages and was slow to change. These three were the earliest Universities in India, dating to 1857, and while only a total of five Universities existed before the start of World War I, there were numerous smaller institutions. The institutional cast of Ramanujan's life included several colleges (Kumbakonam's Government College, Presidency College, Madras Christian College), all with mathematics faculty, often British. The end of the war saw an explosion of universities oriented towards scientific fields, with universities founded in 1916 in Benares, 1918 in Osmania, 1920 in Aligahr, 1921 in Lucknow and Dacca, 1926 in Andhra, and 1929 in Annamalai, all with mathematics departments founded the same or following year (except Dacca, which only had a chemistry and physics department even several decades later). The mathematical-science orientation is apparent from 1921 figures from Calcutta. Out of 174 science graduate, 48 were in physics, 40 in applied mathematics, 62 total in chemistry and applied chemistry, 12 in physiology, and less than ten each in geology, zoology and botany. While these institutions were not necessarily properly funded, there was a full institutional presence of British higher education in every Indian urban center.<sup>154</sup> Of course Ramanujan did not benefit from a structured university course; but as was the case in Cambridge, the university

<sup>&</sup>lt;sup>154</sup> David Arnold, *Science, Technology and Medicine in Colonial India* (Cambridge: Cambridge University Press, 2000), 191-192.

setting was more important as a nexus of mathematical expertise, with actual learning occurring more by informal encouragement, individual tutoring and private study than through lectures.

Until this point, we have encountered several members of the Indian Mathematical Society: Seshu Iyer, Ramachandra Rao, Ramaswami Iyer, and Narayana Iyer. As we can see, active Indian mathematicians spanned the range from amateur enthusiasts to those holding M.A. degrees in mathematics. Professionally, they were either in education or the civil service. Seshu Iyer (1872-1935) received a B.A. from Madras Christian College, but quit before earning an M.A. degree due to ill health. For a time he was a high school Headmaster, then he was a lecturer at Government College (when he was Ramanujan's instructor), from where he moved to Presidency College in Madras, lecturing for 15 years and eventually becoming a Professor of Applied Mathematics.<sup>155</sup> Ramachandra Rao was educated at Presidency College, Madras, but joined the civil service at age 19 and spent the rest of his life there (when Ramanujan met him, he was district collector of a town near Madras) - so he seems to have pursued mathematics in an amateur capacity.<sup>156</sup> Ramaswami Iyer, the founder of the Indian Mathematical Society, also attended Presidency College, Madras, where he received a B.A. and M.A. While there, he contributed to mathematical journals. After, he taught at colleges for some five years before joining the civil service in 1898, rising to Deputy Collector in 1901. Narayana Iyer, as we saw previously, was educated in mathematics but was serving in the civil service in an engineer's role. Hence, for Ramanujan to wind up working in the civil service was not at all unimaginable for a serious mathematician, and even his lack of educational credentials was not unprecedented for

<sup>&</sup>lt;sup>155</sup> Letters and Commentary, 158.

<sup>&</sup>lt;sup>156</sup> Kanigel.

the mathematical community of Indians, though Ramanujan's lack of even a college degree was perhaps the most extreme. These varied individuals had come together to try and form a mathematical community in the form of the Indian Mathematical Society; the community was very much in the process of defining itself and expanding, it was not yet a professional society whose members had a specific social profile. Hence, there was very much a place here for Ramanujan. In fact, it is conceivable that Ramanujan could have remained a mathematical hobbyist employed in the civil service, following in the patterns of these other individuals, and rising to a prominent place in the Indian Mathematical Society. It would have been nothing like his later collaboration with Hardy, but it was not complete obscurity in which he would have languished.

But at this time, there was an emerging new social pattern for students, and that was to study in England or Scotland.<sup>157</sup> Many of these people became the most prominent figures in public life, including many Indian revolutionaries and advocates of independence, including Gandhi, Jinnah and Nehru.<sup>158</sup> In fact, just before Ramanujan went to India, Nehru had been studying there; Nehru returned after seven years' education in 1912.

Much of the Indian population studying in Britain was due to examinations for the Indian Civil Service, required for the highest posts, being given only in London. Similarly, only barristers trained in England could practice law in Calcutta and Bombay. As an added incentive, bar examinations in England were considered easier than those in India.<sup>159</sup> Government scholarship established in 1868 financed a few, but many students were the children of rich elites

<sup>&</sup>lt;sup>157</sup> Visram 44.

<sup>&</sup>lt;sup>158</sup> Visram 86.

<sup>&</sup>lt;sup>159</sup> Lahiri 7.

sent to experience 'English life' and gain the socially prestigious moniker of *Bilat-Pherat*, the England-visited. There were no censuses taken of student populations, but amalgamating several sources, student population rose dramatically from 160 in 1885, to around 200 in 1890, 300 to 380 in 1894, reaching by one count more than 700 in 1910, but by another estimate as many as 1,000 or 1,200, and by 1913 before Ramanujan arrived, there were 1,700-1,800.<sup>160</sup>

These foreign students joined a growing international population in Britain from around the empire. Visram estimates there were several hundred Indians living in Britain between the turn of the century and the start of World War I in 1914, populating every rank of society. There were sailors and laborers surviving on the margins of society, and Indians serving as nurses and domestic servants. Mixing with English high society were exiled Indian nobility, professionals in medicine, education, law, and other fields, and sportsmen and entertainers.<sup>161</sup>

Indians in Britain during this period existed in a strange purgatory – they were not fully accepted, but neither was racism constant and obvious. In the student experience, paranoia about radicalization and disloyalty, and miscegenation, led to the Lee-Warner Committee of Inquiry in 1907 to explore the Indian student experience. The Committee found an increase of incidents of race prejudice, including offensive remarks, a general sense of unfriendliness from British students, and social exclusion.<sup>162</sup> But many students reported no such experiences, and found their stay to be only positive, and even that English students went out of their way to show hospitality to the foreign imperial subjects. Tensions were also dependent on the political status of India, as the Lee-Warner Committee coincided with political agitation in India in the early

<sup>&</sup>lt;sup>160</sup> Lahiri 4-5, Visram 88

<sup>&</sup>lt;sup>161</sup> Visram 44.

<sup>&</sup>lt;sup>162</sup> Visram 92.

1900s.<sup>163</sup> From 1903, all Indians visiting England in any capacity were required to have identity certificates including family background, affluence, and object of visit. While it was partially aimed at controlling a rising number of destitute Indians, it was the student population who ruined the system by 1913, as the majority of them failed to obtain certificates.<sup>164</sup>

As we had quoted from Chandrasekhar above, Ramanujan in the wider context of Indians of this period was seen not as unique, but a member of an Indian renaissance of sorts, a profusion of Indians rising to prominence. In many areas, Indians had arisen to challenge the British on their own terms. Cornelia Sorabji was a pioneer female academic, becoming in 1889 the first woman to study law at a British university.<sup>165</sup> Ranbindranath Tagore won the Nobel Prize in 1913 (though D.H. Lawrence dismissed as "disgusting" and a "fraud" this "wretched worship-of-Tagore attitude," as European civilization stands higher than anything the East ever dreamed of).<sup>166</sup> But the most important example to consider is the prince, K. S. Ranjitsinhji (1872 - 1933), known as Ranji, who became the most famous cricket player in Britain. Kanigel conjectures that even if Hardy – a lifelong cricket fanatic, with the sport rivaling his dedication to mathematics - had any lingering prejudice against Indians, Ranji would have banished it. Ranji was educated in England since 15, and attended Cambridge. While he was seen as an oddity at first, he quickly took to the life on an English gentleman, and through cricket won acceptance. For nine seasons, from 1895 to 1904, Ranji became legendary playing cricket for

<sup>&</sup>lt;sup>163</sup> Visram 89.

<sup>&</sup>lt;sup>164</sup> Lahiri 5.

<sup>&</sup>lt;sup>165</sup> Visram 95-96. She was also the first woman at Deccan College which she attended in India, and at 18, in a college at Ahmedabad in Gujrat, she the first woman to teach at an all-male college, in addition to many other firsts and obstacles (despite her law education, the bar was not opened to women until 1919). While she did not consider herself a feminist, disdaining "women's rights" types, other Indian women were active in the women's suffragette movement.

<sup>&</sup>lt;sup>166</sup> Visram 49.

Sussex. He appeared on cigarette cards, songs and popular music sang his praise, and an 1899 match against Australia had placards all over London declaring, 'Ranji saves England.' He was the first cricketer to score over 3,000 runs in a year, which he did twice in 1899 and 1900. When he went to Australia for a match as captain of Sussex, he was exempted from the £100 entrance tax on non-whites. Newspapers remarked how he demonstrated not only equality, but "indisputabl[e] superiority" to the Englishman.<sup>167</sup>

#### Conclusion

As S. Chandrasekhar would later remark, Ramanujan was remarkable, but by no means singular. He was part of an Indian renaissance of sorts, living at a time where Indians were challenging their colonial masters on their own terms. While I have focused here on the Indian community as a whole, there were increasing numbers of scientists. One of Ramanujan's friends, K. Ananda Rao, set off for Cambridge only five months after Ramanujan did. He had attended Presidency College, one of the places where Ramanujan had failed out. At Cambridge, Rao received one of two Smith's Prizes in 1918, a coveted award. He was a student of Hardy as well; and Ramanujan and Ananda Rao were able to converse with each other in their native Tamil while there. Ananda Rao returned to India in 1919, also shortly after Ramanujan; here, he became a professor of mathematics at Presidency College, where he continued on to have a distinguished research career. Most interestingly, he was a student of Hardy as well, and went on to have a distinguished research career.

<sup>&</sup>lt;sup>167</sup> Visram 97-99.

<sup>&</sup>lt;sup>168</sup> Letters and Commentary, 260-261.

and yet he is very obscure. As I discussed in the beginning, much of Ramanujan's appeal lies in the drama of his life. But Ananda Rao shows that such drama is not necessary for accomplishment; romanticizing can become a self-inflicted complication. This theme we will visit in the conclusion.

# CONCLUSION: Ramanujan Rediscovered

I have argued that constructivism provides a more insightful picture of mathematics than does realism. Here I present some final thoughts about the distinction between the two, in the context of what Ramanujan can tell us about science and society in a more general sense.

The Nobel Laureate and Fellow of the Royal Society, S. Chandrasekhar, wrote the following of Ramanujan:

It must have been a day in April 1920, when I was not quite ten years old, when my mother told me of an item in the newspaper of the day that a famous Indian mathematician, Ramanujan by name, had died the preceding day; and she told me further that Ramanujan had gone to England some years earlier, had collaborated with some famous English mathematicians, and that he had returned only very recently, and was well known internationally for what he had achieved. Though I had no idea at that time of what kind of a mathematician Ramanujan was, or indeed what scientific achievement meant, I can still recall the gladness I felt at the assurance that one brought up under circumstances similar to my own, could have achieved what I could not grasp. I am sure that others were equally gladdened. I hope that it is not hard for you to imagine what the example of Ramanujan could have provided for the young men and women of those times, beginning to look at the world with increasingly different perceptions.

The fact that Ramanujan's early years were spent in a scientifically sterile atmosphere, that his life in India was not without hardships, that under circumstances that appeared to most Indians as nothing short of miraculous, he had gone to Cambridge, supported by eminent mathematicians, and had returned to India with every assurance that he would be considered, in time, as one of the most original mathematicians of the century – these facts were enough – more than enough – for aspiring young Indian students to break their bonds of intellectual confinement and perhaps soar the way that Ramanujan had.

It may be argued, with some justice, that this was a sentimental attitude: Ramanujan represents so extreme a fluctuation from the norm that his being born and Indian must be considered to a large extent as an accident. But to the Indians at the time, Ramanujan was not unique in the way we think of him today, [but] was one of others [like]... Gandhi, Motilal and Jawaharlal Nehru, Rabindranath Tagore...

I think it is fair to say that almost all mathematicians who reached distinction during the three or four decades following Ramanujan were directly or indirectly inspired by his example.<sup>169</sup>

Here, we see the tremendous inspirational potential of a realist account of Ramanujan. Ramanujan's uniqueness, and miraculous transcendence of the retarding social factors, provided an inspiration for the young Chandrasekhar.

Earlier I discussed aesthetics as the ultimate arbiter between realism and constructivism. Here I want to elaborate on that point and say that realism, despite its soaring and lofty nature, condemns us to forever strive for a pure realm that we never seem to reach. Realism might have inspired young mathematicians and scientists, but the story is more complicated than realism will allow. Ranganathan gives the following example:

...a false story was rumored. According to it Ramanujan was supposed to have failed in Mathematics. This story gave an easy handle for cynics to decry our University system. It even caused disaster to a remarkably gifted young mathematician who was my student – T Vijayaraghavan. He systematically neglected his legitimate work in his honours class in mathematics, thereby equating himself with Ramanujan...<sup>170</sup>

Unfortunately for that student, this piece of lore is apocryphal; Ranganathan found Ramanujan's records in old school files, which showed he had consistently done extremely well in his mathematics courses. It is not hard to imagine how such a rumor started: Ramanujan had indeed experienced academic failure in subjects other than mathematics. Moreover, institutions of higher education in India had been embarrassingly unable to recognize his enormous merits in

<sup>&</sup>lt;sup>169</sup> S. Chandrsekhar, "On Ramanujan." Remarks made at the Banquet on June 3, 1987. In *Ramanujan: Essays and Surveys.* Bruce C. Berndt, Robert A. Rankin, eds. pp 26-27

<sup>&</sup>lt;sup>170</sup> Ranganathan 20. The origin of this myth seems to be quite early, referenced in letters even before Ramanujan's death.

light of those failures. This story would illustrate the gap between conventional wisdom and Ramanujan's genius even more dramatically had his failure been of a mathematical nature. As Nandy wrote, this would have "[made] his victory over Western mathematics total."<sup>171</sup> But Ramanujan consistently scored extremely high in mathematics--it was in other subjects that he failed.

Given that this story is false, it shows how a misunderstanding of Ramanujan's biography in combination with an attitude of worship, caused unnecessary and unproductive complications in the mathematical career of at least one person, not least in terms of his potential to receive institutional support. But what if it were true? What significance would it have for somebody who aspires to the mathematical stature of Ramanujan? Would it suggest that by failing out of math classes, one could become a Ramanujan? Or, like some form of academic predestination, would a future Ramanujan necessarily fail out of mathematics classes, because jealous and ignorant institutions would persecute any mathematics as transcendent as his? Both these suggestions are fairly ridiculous, and they create an additional challenge that, for all that Ramanujan faced, was not among his obstacles. Ramanujan failed institutionally because of his inability to attend to other subjects and not much besides.

While this particular example is easily debunked, I believe the dangers of romanticizing auto-didacticism and isolation do not stop here. Consider imagistic representations of Ramanujan. There are four surviving photographs, one of which is preferred overwhelmingly. It is this photo that is included in the book-length publication of Hardy's *Twelve Lectures*. This photo also served as the model for Paul Branlund's bust of Ramanujan. It was also used for a

<sup>171</sup> Nandy 108.

commemorative stamp of Ramanujan issued by the Indian government. Finally, a stylized artistic copy of it adorns the cover of Kanigel's book.

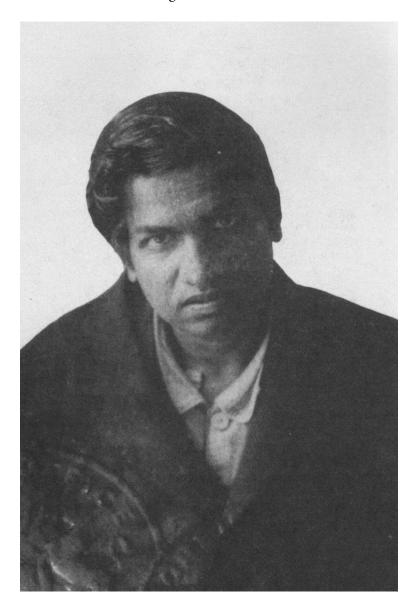


Figure 1. From Bruce C. Berndt, "The Four Photographs of Ramanujan," Essays and Surveys.

Respectively, the bust of Ramanujan, the stamp (pictured as part of the cover of *Letters and Commentary*), and the cover to Kanigel's book:



S. Chandrasekhar wrote a small piece entitled, "On the Discovery of the Photograph of S. Ramanujan, F.R.S."<sup>172</sup> in which he describes the provenance of this image. In 1936, when Hardy was to present his lectures on Ramanujan, he expressed to Chandrasekhar that the only available photograph made Ramanujan look ridiculous. He asked Chandrasekhar if a preferable image was available in India. Chandrasekhar was able to track down Ramanujan's widow, S. Jannaki, who provided him with this passport photo taken in London in 1919, just before Ramanujan returned to India. Hardy's reaction to the photograph was to say, "He looks rather ill (and no doubt he was very ill): but he looks all over the genius he was."

On scientific portraiture, Patricia Fara writes, "because we have inherited Romantic stereotypes of genius, we attribute a thin, pale face, disheveled hair and fine fingers not to melancholy, but to mental brilliance."<sup>173</sup>

Geniuses cannot look ridiculous; they must be tortured and emaciated.

B. M. Wilson, an undergraduate at the time Ramanujan was at Cambridge, reported that Ramanujan would often work for a stretch of 30 hours, and then sleep for 20.<sup>174</sup> Furthermore,

<sup>&</sup>lt;sup>172</sup> Included as a preface to Berndt, *Notebooks* I.

<sup>&</sup>lt;sup>173</sup> Patricia Fara, "Framing the Evidence: Scientific Biography and Portraiture," *The History and Poetics of Scientific Biography*, ed. Thomas Söderqvist, 72.

Ramanujan maintained not only strict vegetarianism, but maintained the Brahmin injunction to only eat food cooked by a fellow Brahmin, which, in most circumstances, meant he had to cook for himself. In one latter, Ramanujan writote, "I have gained a perfect control over my taste and can live on mere rice with a little salad and lemon juice for an indefinite time."<sup>175</sup> We can get a sense for how unrepresentative the iconic picture of Ramanujan is from the fact that one college friend of Ramanujan quoted by Ranganthan said he did not approve of his stamp in the postal picture because Ramanujan's illness made him look dramatically different from how he

I close by revisiting the pictures I include on the cover. The contrast between this picture and the canonical one of Ramanujan highlights what I would like to revisit about him. The Ramanujan of the canonical portrait is lone, tortured and sick; his misery is idealized, romanticized, and glorified. The photograph where he 'looks ridiculous,' in comparison, has him standing in the center, but among colleagues. He does not look particularly joyful in this picture, either; however, this picture represents Ramanujan much more as he appeared for most of his life. It is telling that the picture everyone seems overwhelmingly to prefer is one that shows Ramanujan not as he looked for most of his life, but in the few months preceding his untimely death.

<sup>&</sup>lt;sup>174</sup> R. A. Rankin, "Ramanujan as Patient." *Essays and Surveys*, 57.

<sup>&</sup>lt;sup>175</sup> Letters and Commentary, 125.

<sup>&</sup>lt;sup>176</sup> Ranganathan 75.



Figure 5. From Bruce C. Berndt, "The Four Photographs of Ramanujan," Essays and Surveys.

## References

Because part of my project involves blurring the line between primary and secondary sources, I do not make a separate list here. See my discussion in the Introduction.

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- Ahlfors, Lars Valerian. 1979. Complex analysis : An introduction to the theory of analytic functions of one complex variable. International series in pure and applied mathematics. 3rd ed. New York: McGraw-Hill.

A standard introductory text for complex analysis.

- Aiyar, P. V. Seshu. 1920. The Late Mr. S. Ramanujan, B.A., F.R.S. Journal of the Indian Mathematical Society XII, (3) (Madras: Srinivasa Varadachari & Co., June): 81-86. One of the original obituaries. Elements of this appear in Aiyar and Rao's biography of Ramanujan in the Collected Works.
- Alladi, Krishnaswami. 2003. Review: [RT: Ramanujan : essays and surveys; RA: Berndt, Bruce C. and Robert A. Rankin]. The American Mathematical Monthly 110 (9) (Nov): 861-865.
- Alladi, Krishnaswami. 1996. Review: [RT: Ramanujan : letters and commentary; RA: Berndt, Bruce C. and Robert A. Rankin]. The American Mathematical Monthly 103 (8) (Oct): 708-713.
- Andrews, George E., and Bruce C. Berndt. 2005. Ramanujan's lost notebook. Part I. New York; London: Springer.
- Archibald, Tom. 1993. Review: [RT: The man who knew infinity: A life of the genius Ramanujan; RA: Kanigel, Robert]. Isis 84, (1) (Mar.): 165-6.

This review presents exactly my reaction to Kanigel. Archibald writes, "...a book that presents Ramanujan's life and his mathematics in a balanced way according to reasonable standards of historiography has been desirable for some time. This is not such a book, though in fairness it must be said that it does not claim to be. Robert Kanigel's biography is a well-researched, popular account of Ramanujan's life written in a mass-market journalistic style that many will find more accessible than a scholarly treatment... Certainly much can be learned by reading this account, and documentation (in the form of end notes and a bibliography) is not lacking. The work is therefore potentially quite useful to the historian of mathematics, though to use it will require effort, patience, and caution. Kanigel frequently puts words in the actors' mouths, or thoughts in their minds, and anecdotes for which there is slight documentation are often taken as true." Achibald also comments about the lack of relevant secondary literature – virtually the only work cited is E. T. Bell's *Men of Mathematics*.

Archibald ends by writing, "To the historian of science I recommend the book as excellent in-flight reading. Perhaps it will inspire someone to write the kind of serious monograph the subject deserves."

Aristotle, and J. L. Ackrill. 1987. *A new Aristotle reader* [Selections.]. Princeton, N.J.: Princeton University Press.

For Aristotle's critiques of Plato's Theory of Forms in Metaphysics.

- Arnold, David. 2000. Science, technology, and medicine in colonial India. New Cambridge history of India. Vol. III, 5. Cambridge ; New York: Cambridge University Press.
  A good treatment. Though mathematics is barely mentioned, the book includes some helpful bits of contextual information, such as the founding dates of science departments (pg 191), and descriptions of Indians elected to the Royal Society (pg 194, including S. Chrandrasekhar in 1944 for work in Astrophysics and P. C. Mahalanobis in 1945 for work in Statistics).
- August Entertainment Press Release. 2006. Feature Film on Genius Mathematician Ramanujan : Leading Indian Film Director Dev Benegal and leading British Actor-Director Stephen Fry announce international feature film on genius Indian mathematician Ramanujan. March 10<sup>th</sup>. <a href="http://www.augustentertainment.com/">http://www.augustentertainment.com/</a> , accessed 10/26/08.
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  This is the same E. T. Bell of Bell polynomials; some properties which Bell published in 1934 are found in Chapter 3 of Ramanujan's second (*Notebooks* I, 44). Ramanujan is

mentioned once (pg 328): "For sheer manipulative ability in tangled algebra Euler and Jacobi have had no rival, unless it be the Indian mathematical genius, Srinivasa Ramanujan, in our own century." Note that Bell is not here excluding a treatment of Ramanujan, as Bell ends his book with a treatment of Cantor (1845-1918), which Ramanujan would have come after.

This work is notoriously hagiographical, and much of its content is exaggerated or outright fabricated.

- Bennett, J. A. 1986. The Mechanics' Philosophy and the Mechanical Philosophy. *History of Science* xxiv: 1-28.
- Berndt, Bruce C. 2006. Number theory in the spirit of Ramanujan. Student mathematical library.
  Vol. 34. Providence, R.I.: American Mathematical Society.
  Excellent work at the advanced undergraduate level. This introduces some of
  Ramanujan's work along with presenting the background necessary to understand it, and
  does not presuming the reader is a number theory expert.
- Berndt, Bruce C. 1989. Srinivasa Ramanujan. *The American Scholar* 58 (2) (Spring): 234-244. Biographical treatment by Berndt, although a summary of other sources.
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  Excellent collection of the major essays about Ramanujan, as well as other materials like biographies and descriptions of people close to Ramanujan. A major source.
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- Carr, George Shoobridge. 1886. A synopsis of elementary results in pure mathematics: Containing propositions, formulæ, and methods of analysis, with abridged demonstrations. Supplemented by an index to the papers on pure mathematics which are to be found in the principal journals and transactions of learned societies, both English and foreign, of the present century. London: F. Hodgson; Cambridge, Macmillan & Bowes.
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- Dyson, Freeman J. 1996. Review: [RT: Ramanujan: Letters and commentary; RA: Berndt, Bruce C., and Robert A. Rankin]. *Isis* 87, (2) (Jun.): 387.

A glowing review. Dyson adds an interesting comment: "In recent years, Ramanujanology has become a lively branch of mathematics, with many younger mathematicians exploring the territory that Ramanujan discovered. The advent of computers has brought new talents to he field. A bright graduate student with a Sun workstation can now do almost as good a job as Ramanujan with a pencil."

Galison, Peter Louis. 1997. *Image and logic : A material culture of microphysics*. Chicago: University of Chicago Press.

An influential book, but an example of how mathematics does not seem to fit anywhere in the discipline of history of science. It is not at all apparent how or even if the 'material culture' approach used by Galison would apply to mathematics.

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- ———. 1914. *A course of pure mathematics*. 2d ed. Cambridge Eng.: The University Press. Perhaps more important than any of Hardy's research was this contribution to British mathematical pedagogy. From this introductory text, British students were finally brought up to date with continental analysis.
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  Ist ed. London: Cambridge University Press. Reprinted 1992, Canto ed. Cambridge
  England ; New York: Cambridge University Press.
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I quote some comments made in this as an example of the Indian conception of Ramanujan.

Indian Mathematical Society, The. 1934. Nature (Oct 13): 567.

A short notice to announce a jubilee commemoration issue of the Society's journal. The notice summarizes the Society's history as well.

Jaffrelot, Christophe. 1996. *The Hindu nationalist movement in India*. New York: Columbia University Press.

The most comprehensive account about the history of the Hindu nationalist movement, from its roots in the 1920s. One drawback is that, as this book was published in 1996, it does not get a chance to include some major recent developments, especially the 2002 pogroms against Muslims in the state of Gujrat carried out with the complicity of the regional BJP government.

Kanigel, Robert. 1991. *The man who knew infinity : A life of the genius Ramanujan*. New York : Charles Scribner's Sons; Toronto : Collier Macmillian Canada; New York, Oxford, Singapore, Sydney : Maxwell Macmillian International: Collier Macmillan Canada; Maxwell Macmillan International.

Considered extensively in my thesis, this is the seminal account of Ramanujan.

Kapur, Jagat Narain. 1989. *Some eminent Indian mathematicians of the twentieth century*. New Delhi: Mathematical Sciences Trust Society.

No special content, but Ramanujan is the first person to be treated by this four-volume series, highlighting his importance to a project of this kind.

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Krantz, who knew Erdös and had an Erdös number of one, finds Hoffman's work wanting. He also makes a comment on Kanigel's choice of title. "First, the title of [Hoffman's] book is entirely inappropriate. It could be said of a five-year-old *idiot savant* that he loves only numbers. Paul Erdös was a distinguished mathematician. He helped to establish graph and hypergraph theory; he invented probabilistic method in combinatorics and number theory; he turned Ramsey theory into a full-blown discipline. To say that he 'loved only numbers' is both ridiculous and puerile. One is reminded of the book *The Man Who Knew Infinity* about Ramanujan: Ramanujan was noted for his work in finite mathematics; there is no evidence that he knew much of anything about infinity."

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A fictionalized account of Ramanujan, basically taking the narrative of Ramanujan (including all the most recent scholarship, e.g. *Letters and Commentaries* and *Essays and Surveys*) and filling out imagined dialogue. Surprisingly, Kanigel's (ostensibly nonfiction) account is more evocative, so I doubt this will garner much attention.

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- Littlewood, J. E. 1929. Srinivasa Ramanujan [RT: Collected Papers of Srinivasa Ramanujan]. *Nature* 3104 (123) (27 Apr): 631-3.

Though a review, this is more of a discussion of Littlewood's personal experiences related to Ramanujan. This is the original source of many of Littlewood's poetic and oft-quoted comments about Ramanujan.

Loney, Sidney Luxton. 1895. *Plane trigonometry*. 2d, rev. and enl ed. Cambridge Eng.: University Press.

Merton, Robert King. 1973. *The sociology of science : Theoretical and empirical investigations*. Chicago: University of Chicago Press. Includes a pertinent discussion about the reward systems in science in Chapter 14.

Nature editorial. 2005. Count themselves lucky. Nature 436 (4) (Aug): 603-4.

Argues that despite mathematicians' fear of lack of public respect, they are held in great esteem. To show this, the editorial cites the many popular anecdotes about mathematicians (including Ramanujan and the taxi-cab number 1729 story) and then concludes that mathematics "has haplessly bungled its way into people's hearts." A rather odd piece.

Nature. 1920. Notices of death. Nature 2640 (105) (June 3): 431.

Ramanujan's death first announced in England. He is referred to as "Prof. Srinivasa Ramanujan, F.R.S., fellow of Trinity College, Cambridge;" he had a professorship waiting for him in India, but he was too ill to ever properly assume the post.

University and Educational Intelligence. 1916. Nature 2418 (97) (2 Mar): 24-5.
 Announcement of Ramanujan receiving a B.A. degree by merit of his research.

Nanda, Meera. 2005. *The wrongs of the religious right : Reflections on science, secularism, and Hindutva*. Readings. 1st ed. Gurgaon: Three Essays Collective. Recent reflections, more of the material of her next two listed works.

—. 2003. *Prophets facing backward : Postmodern critiques of science and Hindu nationalism in India*. New Brunswick, N.J.: Rutgers University Press.

A work I had written a research paper around in my junior year. It includes an excellent argument about how the right-wing Hindu nationalist movement has co-opted leftist critiques, though the blame Nanda puts on leftist intellectuals for this is a weaker argument. Though a strongly realist perspective, it is an excellent analysis of some intellectual trends of Hindu nationalism.

—. 2003. Postmodernism and religious fundamentalism : A scientific rebuttal to Hindu science : An essay, a review and an interview. Pondicherry: Navayana.

Interestingly, though Ramanujan had his own pet theories of 'Vedic mathematics' (as Mahalanobis reports), Ramanujan seems never to have had any nationalist inclinations (an issue explored by Nandy). But normally, 'Vedic mathematics' is part of the Hindu nationalist agenda in claiming all knowledge originates in the Vedas, and Nanda here debunks it.

Also includes (pg 24) the following critique of Nandy: "It is not enough, therefore, to scoff at [fundamentalists] as 'another manifestation of the modern mindset', and move on. (This is the strategy of those like Ashis Nandy and other professed gandhian antisecularists who are now scrambling to establish anti-hindutva credentials. After years of haranguing the secularist currents in Indian society, Nandy and other nativists now have the impossible task of distinguishing their own calls for 'alternative modernity' from a fullblooded 'Hindu modernity' favored by hindutva.)"

I am a fan of Nanda's political efforts and intellectual output, but I see realist currents in the absolutism of Hindu nationalism, as in my argument about Ranganathan (though he does not make political statements and therefore cannot be called an outright nationalist, his work certainly fits perfectly into a nationalist agenda). But Nanda's focus on the relativism is not incompatible with this, because it is using the banner of relativism that the Hindu right makes its religious realist claims equal to the dominant, western scientific brand of realism.

Nandy, Ashis. 1980. *Alternative sciences : Creativity and authenticity in two Indian scientists*. Man, state, and society series. Vol. 4. New Delhi: Allied.

One of my major sources. Note the critique by Nanda quoted directly above; my own critique of Nandy, though, is substantially different, because I argue that he (like Nanda!) is a realist.

- Naraniengar, M. T. and S. Narayana Aiyar, eds. *Journal of the Indian Mathematical Club* I (Madras: S. Murthy & Co., The "Kapalee" Press).
  The first few issues of the Journal of the Indian Mathematical Society, at that time still called the Indian Mathematical Club. Includes a list of members, as well as monthly updates of periodicals received and books purchased, for circulation among members.
- Nehru, Jawaharlal. 1946. *The discovery of India*. New York: The John Day company. pp. 215-216.
  Nehru gives the following discussion of Ramanujan: "Mathematics in India inevitably makes one think of one extraordinary figure of recent times. This was Srinivasa
  Ramanujan. Born in a poor Brahman family in south India, having no opportunities for a proper education, he became a clerk in the Madras Port Trust. But he was bubbling over with some irrepressible quality of instinctive genius and played about with numbers and equations in his spare time. By a lucky chance he attracted the attention of a mathematician who sent some of his amateur work to Cambridge in England. People

there were impressed, and a scholarship was arranged for him. So he left his clerk's job and went to Cambridge and during a very brief period there did work of a profound value and amazing originality. The Royal Society of England went rather out of their way and made him a fellow, but he died two years later, probably of tuberculosis, at the age of thirtythree [sic]. Professor Julian Huxley has, I believe, referred to him somewhere as the greatest mathematician of the century.

"Ramanujan's brief life and death are symbolic of conditions in India. Of our millions how few get any education at all, how many live on the verge of starvation; of even those who get some education, how many have nothing to look forward to but a clerkship in some office on pay that is usually far less than the unemployment dole in England. If life opened its gates to them and offered them food and healthy conditions of living and education and opportunities of growth, how many among these millions would be eminent scientists, educationists, technicians, industrialists, writers and artist, helping to build a new India and a new world?"

Neville, E. H. 1948. Mathematics in India [RT: Ancient Indian Mathematics and Vedha; RA: Gurjar, Prof. L. V.]. *Nature* 4094 (161) (17 Apr): 580-1.

Neville here identifies this work as driven by Hindu nationalism. He critiques its attempts to uncritically argue for granting India priority for many discoveries of the past. Because Neville was such an important figure in supporting Ramanujan, it is interesting here to note that for him, Ramanujan's greatness has not extrapolated into sympathy of any Hindu nationalist projects trying to historically win prestige for India in science.

–. 1942. Srinivasa Ramanujan. *Nature* 3776 (149) (14 Mar): 292-5.

Transcript of a talk broadcast to India in 'Hindustani' (presumably, Hindi) on April 22, 1941.

—. 1924. Srinivasa Ramanujan. *Nature* 2838, (113) (22 March 1924): 426. A correction about Ramanujan's birth date.

—. 1921. The late Srinivasa Ramanujan. *Nature* 2673, (106) (20 January 1921): 661-2. Hardy and Neville seem to have circulated obituaries of Ramanujan primarily within the journal *Nature*, at least for the western press. Hardy's original obituary lacked biographical information, which Neville provides here.

Nye, Mary Jo. 2006. Scientific biography: History of science by another means? *Isis* 97, (2) (06/15): 322-9.

Plato, and Francis Macdonald Cornford. 1959. *Plato's Timaeus*. The library of liberal arts. [Timaeus.]. Vol. 106. New York; London: Macmillan; Collier Macmillan. For the source of 'Platonism' as an idea and as a term.

Porter, Theodore M. 2006. Is the life of the scientist a scientific unit? Isis 97, (2) (06/15): 314-21.

- Putnam, Hilary. 1979. *Mathematics, matter, and method*. His philosophical papers. 2d ed. Vol. 1. Cambridge ; New York: Cambridge University Press. A statement of a developed philosophical position of realism.
- Putnam, Hilary, and Paul Benacerraf. 1983. *Philosophy of mathematics : Selected readings*. 2nd ed. Cambridge Cambridgeshire ; New York: Cambridge University Press. An influential collection of essays that I use for background.
- Raina, Dhruv. 2003. *Images and contexts : The historiography of science and modernity in india*. New Delhi: Oxford University Press.

Though I have not used this much, it is an excellent source. Includes as discussion of Ramanujan and Hardy (pgs 165-8) especially in the context of Shils' article (cited above). Raina (pg 167) correctly identifies the constructivist problem as "how a mathematician at the periphery could invent a mathematics that surpassed insights available at the centre." I would reply, of course, that there were ties between the periphery and center that have heretofore not been properly emphasized.

- Rajagopalan, K. R. 1988. *Srinivasa Ramanujan*. Madras: Sri Aravinda-Bharati. A tiny volume, it is out-and-out hagiographical, conspicuously shows its total lack of understanding of mathematics, and lacks citations, all making it of extremely questionable validity. I discuss this briefly in my introduction.
- Rao, Dewan Bahadur R. Ramachandra. 1920. In Memoriam: S. Ramanujan. *Journal of the Indian Mathematical Society* XII, (3) (Madras: Srinivasa Varadachari & Co., June): 87-90. One of the original obituaries. Elements of this appear in Aiyar and Rao's biography of Ramanujan in the *Collected Works*.
- Rao, K. Srinivasa. 1993. *Srinivasa Ramanujan: a mathematical genius*. Madras: EW Books Pvt. Ltd.

A recent account. K. Srinivasa Rao seems to be making a tremendous effort to establish himself as the leading current Ramanujan expert in India. His work contains no new information, and reads very similar to Ranganathan. However, Rao's website < http://www.imsc.res.in/~rao/ramanujan/> is a tremendous resource. Rao posts scans of large amounts of work relating to Ramanujan, including all of his *Collected Papers*, presented one page scan at a time, the entirety of the facsimile pages of his notebooks presented the same way. The site also includes excerpts from other works, transcripts of Rao's interviews (with people like Berndt, Rankin, and the grandchildren of Narayana Iyer), pictures, and much else. I should add that the posted scans are of dubious legality, as most of these works have been recently reprinted. But I hope that he can get away with it being in India, as making this content so freely available is a great service.

Ramanujan Aiyangar, Srinivasa. 1957. *Notebooks*. Bombay: Tata Institute of Fundamental Research.

Facsimile copies of Ramanujan's three notebooks, published in two huge volumes.

Ramanujan Aiyangar, Srinivasa, and Bruce C. Berndt. 1985. *Ramanujan's notebooks*. Parts I-V. New York: Springer-Verlag.

The end result of Bruce Berndt's monumental two-decade long project to edit (providing complete proofs) all the content of Ramanujan's notebooks.

Ramanujan Aiyangar, Srinivasa, Bruce C. Berndt, and Robert A. Rankin. 1995. *Ramanujan : Letters and commentary*. History of mathematics. [Correspondence.]. Vol. 9. Providence, R.I.: American Mathematical Society.

Collection of all available primary sources related to Ramanujan, retyped (sometimes translated from Tamil) and with commentary providing background information about the writers or addressees, pointing out mistakes in the letters, and giving mathematical comments when relevant.

- Ramanujan Aiyangar, Srinivasa, G. H. Hardy, P. V. Seshu Aiyar, and B. M. Wilson. 2000. *Collected papers of Srinivasa Ramanujan*. Providence, RI: AMS Chelsea Pub.
  Reprint of the work published shortly after Ramanujan's death to commemorate him. The biography given here is the first written about Ramanujan. It was the authoritative account for many decades, and remains a major source for all subsequent biographies. However, the information contained here is often printed earlier by Aiyar or Rao in newspaper notices (see *Letters and Commentary*) or obituaries (in the JIMS, cited here).
- Ramanujan Aiyangar, Srinivasa, and George E. Andrews. 1988. *Ramanujan revisited : Proceedings* of the centenary conference, university of Illinois at Urbana-Champaign, June 1-5, 1987.

Boston: Academic Press.

Mostly mathematics, but the first two essays are of more general relevance. Fortunately, they are both reprinted in *Essays and Surveys*.

- Ranganathan, S. R. 1967. *Ramanujan, the man and the mathematician*. Great thinkers of India series. 1st ed. Vol. 1. Bombay, New York: Asia Pub. House. One of my major sources, discussed extensively.
- Richards, Joan L. 2006. Introduction: Fragmented lives. Isis 97, (2) (06/15): 302-5.
- Shapiro, Stewart. 2005. *The oxford handbook of philosophy of mathematics and logic*. Oxford handbooks in philosophy. Oxford ; New York: Oxford University Press. Background for philosophy of mathematics.
- Shils, Edward. 1991. Reflections on tradition, centre and periphery and the universal validity of science: the significance of the life of S. Ramanujan. *Minerva* 29 (4) (Dec): 391-419.
  A realist polemic against constructivism that mobilizes Ramanujan as an example. Shils makes standard arguments of the 'Science Wars.' For example (pg 400): "The main significance of the [Strong] programme lies in its being part of a wide intellectual movement to derogate civil or bourgeois society, democratic liberalism, Western civilisation as a whole and science as one of the greatest achievement of that civilisation." Shils writes (pg 407), "From thinking about Ramanujan, I have concluded that there are no territorial or social or religious or ethnic limitations on the validity of what a scientist discovers." To me it seems like mobilizing Ramanujan in this manner is an extremely obvious move for a realist. It surprises me that it is not common, let alone that I have not found this done elsewhere else.
- Söderqvist, Thomas. 2007. *The history and poetics of scientific biography* : *Science, technology and culture, 1700-1945*. Aldershot, England ; Burlington, VT: Ashgate. Excellent collection of essays relating to scientific biography. While few of the essays are directly theoretical, I apply the comments and observations made around specific case studies to my own case.
- Soddy, Frederick. 1941. Qui s'accuse s'acquitte [RT: A mathematician's apology; RA: Hardy, G.H.]. *Nature* 3714 (147) (Jan): 3-5.

A brutal and incisive attack on Hardy's work, but it seems the two were friends and as the antagonism is probably playful. Sokal, Alan D., and J. Bricmont. 1998. *Fashionable nonsense : Postmodern intellectuals' abuse of science*. 1st Picador USA ed. New York: Picador USA.

This book is an important realist critique about the role and use of mathematics in the history of science. But I believe my thesis does not fall under any of its criticisms, as my thesis neither uses mathematics as a bank of metaphors, nor does its normative recommendations for mathematics derive from anything outside of the study of how mathematics works. While I think they misunderstand relativism and constructivism, nonetheless my use of these is not essentialist, but contingent on methodological fruitfulness.

Srinivasan, P. K. 1967. *Ramanujan : An inspiration (Junior mathematics number).* Ramanujan memorial number, Volume II. Madras : The Muthialpet High School Number Friends Society Old Boy's Committee.

This is a collection of essays, most of them of an elementary mathematical nature, some of which are in Tamil. It is the second of two volumes by Srinivasan, both of which are very obscure. I had trouble tracking this down, and unfortunately was not able to find "Ramanujan memorial number volume one," which is Srinivasan's collection of primary source material. Though I would have preferred to be able to compare the two, Srinivasan's material is apparently reprinted in *Letters and Commentary*, including additional material Srinivasan had not published.

Steptoe, Andrew. 1998. *Genius and the mind : Studies of creativity and temperament*. Oxford ; New York: Oxford University Press.

Chapter 6 is an explicit treatment of Ramanujan as a genius by Robert S. Albert, "Mathematical giftedness and mathematical genius: a comparison of G. H. Hardy and Srinivasa Ramanujan." This offers no new research, only interpretation, so I do not deal with it in the body of my paper. Albert tries to describe the life of Ramanujan, a genius, with the life of Hardy, who was gifted but not a genius, to try and identify some social or psychological trends to identify gifted or genius children. He has extensive charts and graphs comparing publication output to age, publication output as a measure of eminence, ranking of Hardy and Ramanujan on a set of 33 personality scales, creativity and eminence by profession and gender. Many of the measures are arbitrary or based on questionable concepts, the sample sets narrow, and the results ambiguous. Albert takes the historical narrative provided by Kanigel, Ranganathan, Nandy and others for granted.

An interesting comment made by Andrew Steptoe in the introductory chapter is that, "It is necessary for most of us to take on trust the fact, for example, that the Indian

mathematician Srinivasa Ramanujan discussed by Robert Albert in Chapter 6 was a genius, since it requires advanced mathematical training to understand his field of activity." These are exactly the assumptions I question.

Summerhayes, V. S. 1948. Obituaries: Prof. G. H. Hardy, F.R.S. *Nature* 4099 (161) (22 May): 797-9.

Obituary giving some good information about Hardy's place and importance in British mathematics.

Terrall, Mary. 2006. Biography as cultural history of science. *Isis* 97, (2) (06/15): 306-13.

- Tomlin, Sarah. 2005. What's the plot? Can mathematicians learn from the narrative approaches of the writers who popularize and dramatize their work? *Nature* (436) (4 Aug): 622-3. Mentions the play *Partition* by Ira Hauptman, loosely based on G. H. Hardy and Ramanujan.
- Van Fraassen, Bas C. 1980. *The scientific image*. Clarendon library of logic and philosophy. Oxford; New York: Clarendon Press; Oxford University Press.
- Van Sant, Gus, Robin Williams, Matt Damon, Ben Affleck, Stellan Skarsgård, Minnie Driver, Miramax Films, Miramax Home Entertainment, and Buena Vista Home Entertainment.
   1997. Good Will Hunting. Miramax collector's series. Widescreen version ed. Burbank, Calif.: Miramax Home Entertainment : Distributed by Buena Vista Home Entertainment.

I transcribe a dialogue about Ramanujan in a footnote.

- Visram, Rozina. 2002. *Asians in Britain : 400 years of history*. London ; Sterling, Va.: Pluto Press. Excellent secondary source, of far wider scope than Lahiri.
- Warwick, Andrew. 2003. *Masters of theory : Cambridge and the rise of mathematical physics*. Chicago: University of Chicago Press. One of my major sources for theory.
- Young, D. A. B. 1994. Ramanujan's illness. *Notes and Records of the Royal Society of London* 48, (1) (Jan.): 107-19. Reprinted in *Essays and Surveys*.

Zucker, I. J. 1994. Review: India's mathematical prodigy [RT: The man who knew infinity : A life of the genius Ramanujan; RA: Kanigel, Robert]. *Notes and Records of the Royal Society of London* 48, (2) (Jul.): 325-6.

A very positive review, although Zucker does add one qualification, writing "Though Kanigel has explored his early life thoroughly, no hard information emerges as to how an untutored boy from South India was able to tackle mathematical problems of alarming complexity, nor are any clues offered as to how he was able to formulate such questions in the first place."